

PERT and CPM

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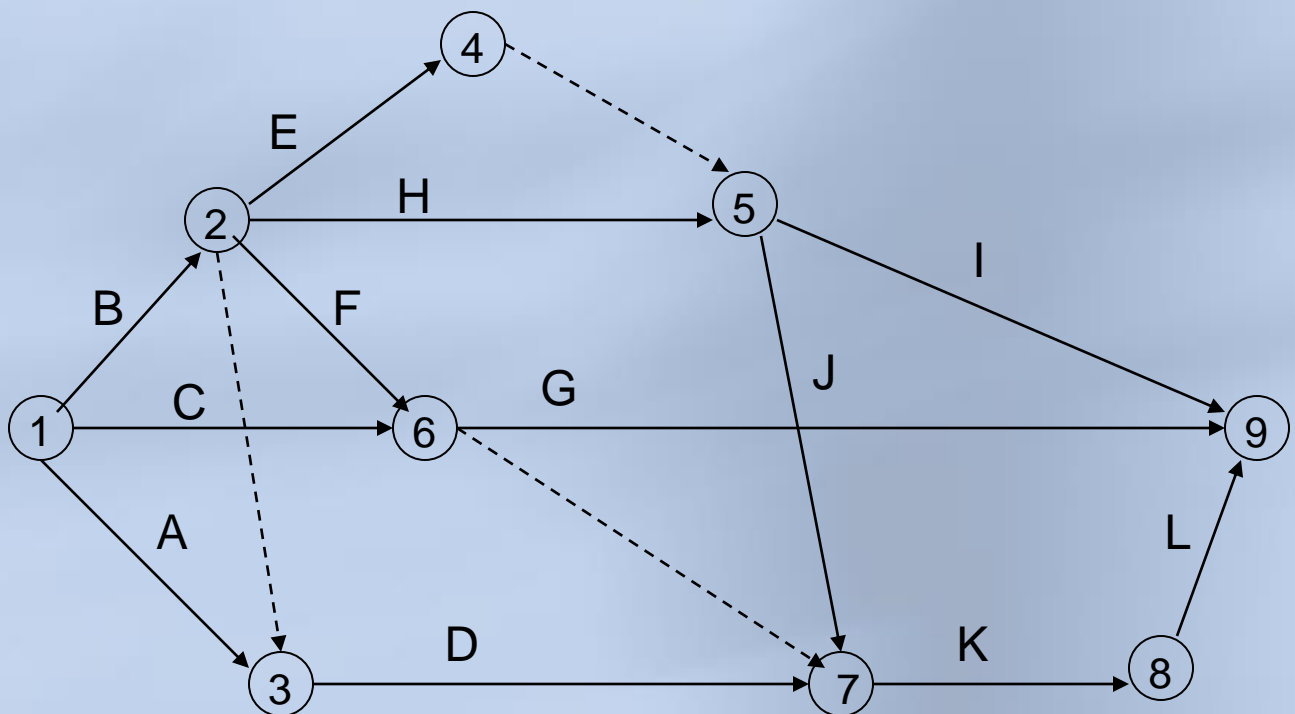
Rules of Network Construction

1. Each activity is represented by one arrow
2. An activity can begin only when all its predecessors are done
3. Length and bearing of arrows is of no consequence
4. Arrow direction indicates general progression in time – tail events represent start while head events represent end of activities
5. Events are identified by numbers while activities are represented by their starting and ending events
6. A network should have only one initial and one final node
7. Introduce as few dummy activities as may be necessary
8. Looping is not permitted

Network

Activity	Imm. Pred.
A	-
B	-
C	-
D	A, B
E	B
F	B

Activity	Imm. Pred.
G	C, F
H	B
I	E, H
J	E, H
K	C, D, F, J
L	K

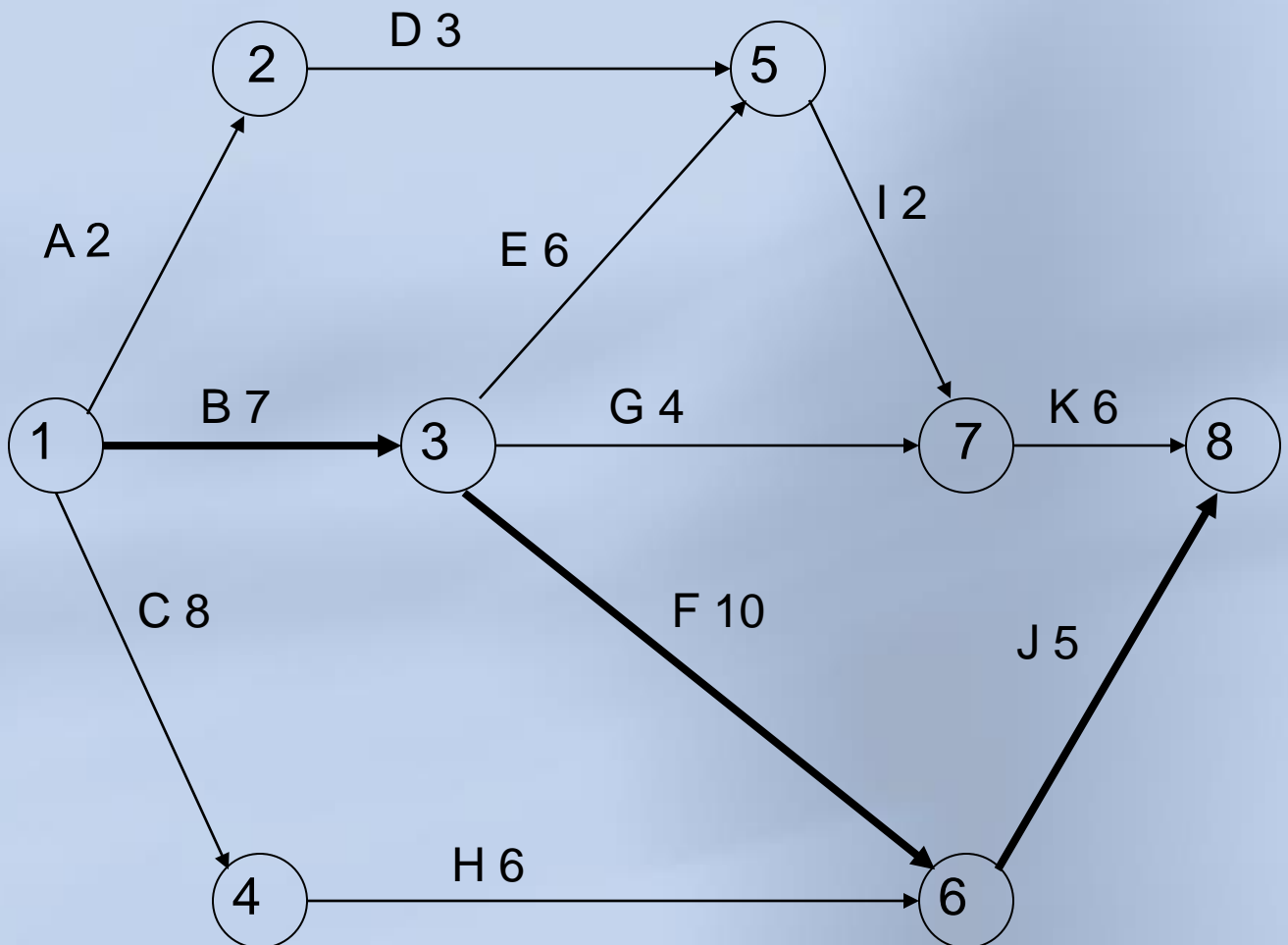


Critical Path

- Critical path refers to the longest path of a given project network
- Duration of a project is given by the length of the critical path
- Activities on a critical path are called critical activities while remaining activities are non-critical
- A project can have more than one critical path as well
- Critical activities are so called because their timely completion is critical to the completion of the project in time
- Critical activities can not be delayed while non-critical activities have some cushion available

Critical Path (...continued)

- **Critical Path: 1-3-6-8**
- **Critical Activities: B F J**
- **Project Duration: 22 days**
- **Non-critical Activities: A C E G H I K**



Floats

- **Total float** is the amount of time by which an activity may be delayed without delaying the project completion
Caution: interpret total floats of activities carefully - all can not be used independently
- **Free float** is that part of total float which can be used without affecting floats of the succeeding activities
- The part of total float which is not free is called **interfering float**
- **Independent float** is the amount of time which can be used without affecting the head and the tail events
Total Float \geq Free Float \geq Independent Float

Calculation of Floats

- **Total float**

= Latest start time of the activity – Earliest start time of the activity

- **Free float**

= Total float – Head event slack

= Earliest start time of the next activity – Earliest completion time of the activity

- **Interfering float**

= Total float – Free float

- **Independent float**

= Earliest start time of the next activity – Latest finish time of the preceding activity – Duration of the activity

= Free float – Tail event slack, or zero, whichever is higher

Floats Calculation

i - j	t	ES	EF	LS	LF	TF	IF	FF	InF
1-2	2	0	2	9	11	9	9	0	0
1-3	7	0	7	0	7	0	0	0	0
1-4	8	0	8	3	11	3	3	0	0
2-5	3	2	5	11	14	9	1	8	0
3-5	6	7	13	8	14	1	1	0	0
3-6	10	7	17	7	17	0	0	0	0
3-7	4	7	11	12	16	5	1	4	4
4-6	6	8	14	11	17	3	0	3	0
5-7	2	13	1	14	16	1	1	0	0
6-8	5	17	22	17	22	0	0	0	0
7-8	6	15	21	16	22	1	0	1	0

Resource Analysis and Allocation

Problems related to resources include

- **Time-cost trade-off**, called *crashing*
Completion time of the project is systematically reduced considering cost
- **Resource Levelling**: An attempt is made to smoothen the resources requirements by systematically scheduling the activities
- **Resource Allocation**: Allocation of limited availability of resources in a manner that the project is completed in minimum time

Crashing

- Involves use of additional resources with a view to reduce the project duration
- Crashing increases direct cost of doing activities but reduces overhead, which is usually a function of time
- It may also help to reap any benefits of early completion
- It needs information on cost of reduction of every activity per unit of time
- In crashing, always consider the critical path and choose the activity that can be crashed at least cost
- In case of multiple critical paths, all of them should be considered for crashing simultaneously
- Time-cost schedule is prepared to get information on normal time and the cost, the minimum time to complete project and the cost, and optimal duration of the project

Resource Leveling

- Resource leveling refers to scheduling the activities of a project in such a manner that the requirement of resources may be leveled to the extent possible without delaying the completion of the project
- To begin with every activity is scheduled at the earliest and the requirement of the resource involved is assessed on a daily basis
- Since the activities are scheduled on early days, the demand for the resource, say labour, is likely to be large in earlier days of the project and small in the later days

Resource Leveling

(...continued)

- Attempt is then made to shift the performance of non-critical activities depending on the floats available so that demand for the resource is reduced somewhat in the early days and increased in later days. Adjustments are made in such a way that the resource demand becomes as uniform as possible
- Thus, attempt is made to level the resource demand without adversely affecting the project duration

PERT

- Uses three time estimates for every activity: *optimistic* (a), *most likely* (m) and *pessimistic* (b)
- They are used to obtain expected times and their variances

Expected time = $(a+4\times m+b)/6$;

and Variance = $[(b - a)/6]^2$

- Obtain critical path using expected times and identify critical activities
- Add variances for critical activities to get variance for completion time and find its square root to get standard deviation
- With expected duration and standard deviation, calculate probabilities of project completion using normal distribution

PERT Calculations

Example 12.9

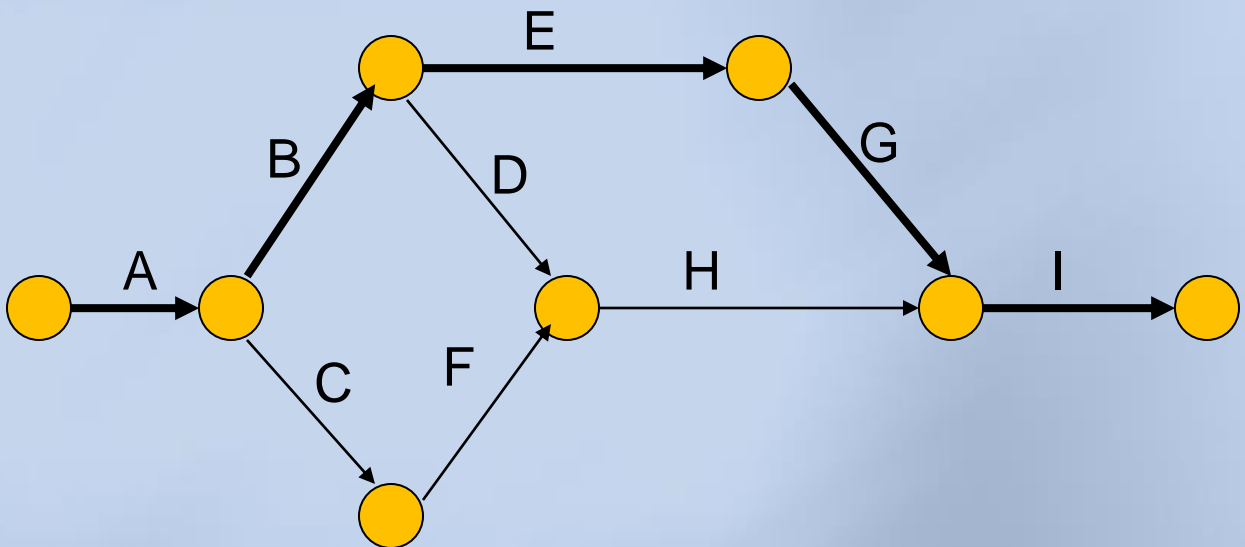
Activity	a	m	b	t_e	σ^2
A	4	6	8	6	4/9*
B	5	7	15	8	25/9*
C	4	8	12	8	16/9
D	15	20	25	20	25/9
E	10	18	26	18	64/9*
F	8	9	16	10	16/9
G	4	8	12	8	16/9*
H	1	2	3	2	1/9
I	6	7	8	7	1/9*

- Critical activities: A B E G I
- Project duration = $6+8+18+8+7 = 47$ days
- Project variance = $4/9 + 25/9 + 64/9 + 16/9 + 1/9 = 110/9$
- Project standard deviation = $\sqrt{110/9} = 3.496$

PERT Calculations

(...continued)

Project Network



- For Pr (completion in 55 days): $Z = (X - \mu)/\sigma$
 $Z = (55 - 47)/3.496 = 2.29$. Now, Area to the left of $Z = 2.29$ is $0.5 + 0.4890 = \mathbf{0.9890}$
- For Pr (completion with 0.90 chance):
 Z corresponding to area 0.40 (between μ and X) is 1.28. Thus, $1.28 = (X - 47)/3.496$ and $X = 51.47$ or 52 app.
- The project should start 52 days before due date

PERT/Cost

- A modification of PERT which allows planning, monitoring and controlling of project cost and duration
- Budgeting process allows to determine how much money is to be spent every time unit (week/month) over planned duration of project
- Budgets are prepared on the basis of earliest and latest start times of activities
- Comparisons are made continuously between the actual costs and budgeted amounts
- Comparisons are also made between percentage of work done and percentage of budgeted amount spent: if former is smaller than the later, we have *budget overrun*

Multiple Choice Questions

● CPM is:

1. Critical Project Management.
2. Critical Path Management.
3. Critical Path Method.
4. Crash Project Method.

Multiple Choice Questions

- Which of these is not correct?
 1. PERT is probabilistic in nature.
 2. CPM is deterministic in nature.
 3. CPM is event-oriented.
 4. CPM and PERT use similar terminology, but were developed independently.

Multiple Choice Questions

● Mark the wrong statement:

1. A project is a set of activities, which can be performed in a certain logical sequence.
2. A network is a graphic portrayal of interdependency relationship among the activities of a project.
3. An arrow representing an activity can have any length and shape.
4. An activity cannot be represented by more than one arrow but an arrow can represent one or more activities.

Multiple Choice Questions

● Point out what is not required. While considering precedence relationships,

1. all the predecessor(s) of an activity should be focussed on.
2. only immediate predecessor(s) should be focussed on.
3. redundant predecessors should be dropped.
4. care must be taken that there is no logical fault it as may result in formation of a loop.

Multiple Choice Questions

● Mark the correct statement:

1. The head event of an activity also represents tail event of its predecessor activity.
2. Nodes should be numbered in such a way that, for every activity, the initial node bears a lower number while final node bears a higher number.
3. The event marking start of an activity is called head event and the event marking the end is termed as tail-event.
4. In drawing a network, it may happen that A precedes B, B precedes C and C precedes A.

Multiple Choice Questions

- Which of the following is not a rule of network construction?
 1. Each defined activity is represented by one and only one arrow.
 2. A network should have only initial and one terminal node.
 3. Identical initial and final nodes can identify two activities.
 4. Only as few dummy activities should be included as is warranted.

Multiple Choice Questions

● Mark the wrong statement:

1. Forward pass calculations yield the earliest and latest start & finish times of various activities.
2. The difference between latest and earliest finish times of an activity is its total float.
3. Free float of an activity cannot exceed its total float.
4. Determination of earliest and latest start time of various activities of a project is useful for proper planning of their execution.

Multiple Choice Questions

- It is known that in a project, an activity 4-6 has duration of 6 days and total float of 3 days. The E and L times at node 4 are 8 and 11 respectively while at node 6, both are equal to 17. Which of the following is not a true statement about 4-6?

1. Its free float is 3 days.
2. Its independent float is 0.
3. It is a critical activity.
4. The ES of this activity is 8.

Multiple Choice Questions

● Mark the wrong statement:

1. All activities on a critical path are critical activities.
2. A project network may have none, one or more critical paths.
3. A delay in a critical activity surely delays the completion of project.
4. Each critical activity has identical earliest and latest start times.

Multiple Choice Questions

● Pick the wrong relationship:

1. Interfering Float = Total Float – Free Float.
2. Total Float = Free Float + Independent Float.
3. Total Float \geq Free Float \geq Independent Float.
4. Free Float = Total Float – Head Event Slack.

Multiple Choice Questions

- Which of the following is not correct in respect of PERT calculations?
 1. Expected time of an activity is a weighted average of three time estimates, a , m , and b with respective weights of 1, 4 and 1.
 2. The completion time of an activity is assumed to follow normal distribution.
 3. The standard deviation of activity completion time is one-sixth of the excess of pessimistic over optimistic time.
 4. The sum total of variances of critical activity times gives the variance of the overall project completion time.

Multiple Choice Questions

- Given, expected duration of project = 47 days, variance = 9 days. What is the probability of not completing the project in 50 days?

Z	0.33	1.00
Area	0.1293	0.3413

- 0.6293
- 0.8413
- 0.1587
- 0.3707

Multiple Choice Questions

- In the previous Question, what is the probability of completing the project within 3 days more or less than the expected time?

1. 0.5000

2. 0.6826

3. 0.2586

4. 0.3174

Multiple Choice Questions

- Which of the following is not true about crashing?
 1. Always focus on critical path and choose between critical activities only.
 2. Crashing cost per day = $(\text{Crash cost} - \text{Normal cost}) / (\text{Normal time} - \text{Crash time})$.
 3. Of all activities of the project, choose the activity with least crashing cost.
 4. If there are multiple critical paths, crash them all.

Multiple Choice Questions

● Mark the wrong statement:

1. An event, which represents the initiation of more than one activity, is called the burst event.
2. The longest path of a given project gives the maximum duration while its shortest path indicates shortest duration.
3. The resource allocation programmes aim to allocate the given resources in a manner that the project completes in minimum time.
4. A non-critical activity may or may not have any free float.