Introduction

- Body of Knowledge
- Problem Solving and Decision Making
- Quantitative Analysis and Decision Making
- Quantitative Analysis
- Models of Cost, Revenue, and Profit
- Quantitative Methods in Practice

Body of Knowledge

- The body of knowledge involving quantitative approaches to decision making is referred to as
  - Management Science
  - Operations Research
  - Decision Science
- It had its early roots in World War II and is flourishing in business and industry due, in part, to:
  - numerous methodological developments (e.g. simplex method for solving linear programming problems)
  - a virtual explosion in computing power

Problem Solving and Decision Making

- 7 Steps of Problem Solving

  (First 5 steps are the process of decision making)

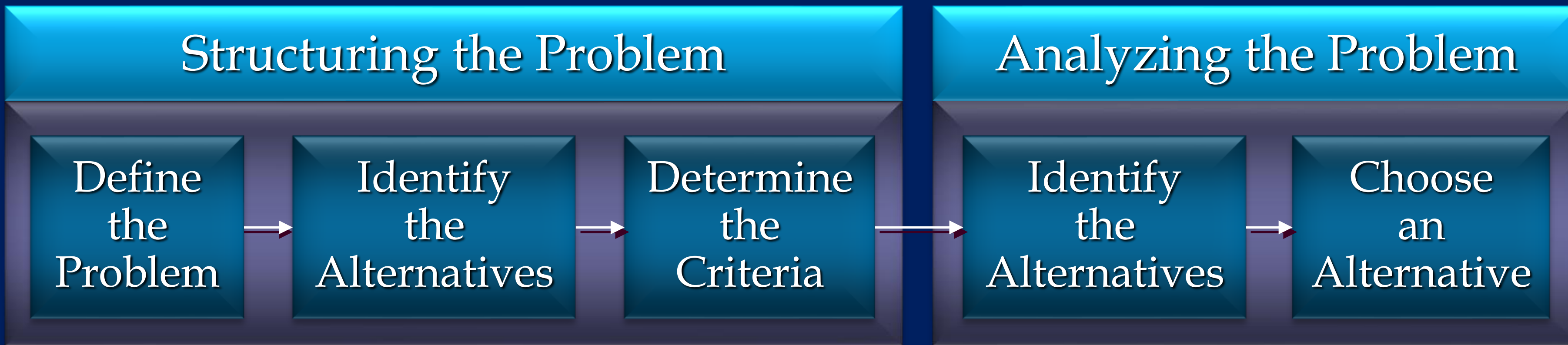
  1. Identify and define the problem.
  2. Determine the set of alternative solutions.
  3. Determine the criteria for evaluating alternatives.
  4. Evaluate the alternatives.
  5. Choose an alternative (make a decision).

  ----

  6. Implement the selected alternative.
  7. Evaluate the results.

Quantitative Analysis and Decision Making

Decision-Making Process



- Problems in which the objective is to find the best solution with respect to one criterion are referred to as single-criterion decision problems.
- Problems that involve more than one criterion are referred to as multicriteria decision problems.

Quantitative Analysis and Decision Making

- Analysis Phase of Decision-Making Process

  Qualitative Analysis

  - based largely on the manager's judgment and experience
  - includes the manager's intuitive "feel" for the problem
  - is more of an art than a science

Quantitative Analysis and Decision Making

- Analysis Phase of Decision-Making Process

  Quantitative Analysis

  - analyst will concentrate on the quantitative facts or data associated with the problem
  - analyst will develop mathematical expressions that describe the objectives, constraints, and other relationships that exist in the problem
  - analyst will use one or more quantitative methods to make a recommendation

Quantitative Analysis and Decision Making

- Potential Reasons for a Quantitative Analysis Approach to Decision Making
  - The problem is complex.
  - The problem is very important.
  - The problem is new.
  - The problem is repetitive.

Quantitative Analysis

- Quantitative Analysis Process
  - Model Development
  - Data Preparation
  - Model Solution
  - Report Generation

Model Development

- Models are representations of real objects or situations
- Three forms of models are:
  - Iconic models - physical replicas (scalar representations) of real objects
  - Analog models - physical in form, but do not physically resemble the object being modeled
  - Mathematical models - represent real world problems through a system of mathematical formulas and expressions based on key assumptions, estimates, or statistical analyses

Advantages of Models

- Generally, experimenting with models (compared to experimenting with the real situation):
  - requires less time
  - is less expensive
  - involves less risk
- The more closely the model represents the real situation, the more accurate the conclusions and predictions will be.

Mathematical Models

- Objective Function – a mathematical expression that describes the problem's objective, such as maximizing profit or minimizing cost
    - Consider a simple production problem. Suppose x denotes the number of units produced and sold each week, and the firm's objective is to maximize total weekly profit. With a profit of \$10 per unit, the objective function is $10x$.

Mathematical Models

- Constraints – a set of restrictions or limitations, such as production capacities
  - To continue our example, a production capacity constraint would be necessary if, for instance, 5 hours are required to produce each unit and only 40 hours are available per week. The production capacity constraint is given by $5x \leq 40$.
  - The value of $5x$ is the total time required to produce x units; the symbol $\leq$ indicates that the production time required must be less than or equal to the 40 hours available.

Mathematical Models

- Uncontrollable Inputs – environmental factors that are not under the control of the decision maker
  - In the preceding mathematical model, the profit per unit ($10), the production time per unit (5 hours), and the production capacity (40 hours) are environmental factors not under the control of the manager or decision maker.

Mathematical Models

- Decision Variables – controllable inputs; decision alternatives specified by the decision maker, such as the number of units of a product to produce.
  - In the preceding mathematical model, the production quantity x is the controllable input to the model.

Mathematical Models

- A complete mathematical model for our simple production problem is:

Maximize	$10x$	(objective function)
subject to:	$5x \leq 40$	(constraint)
	$x \geq 0$	(constraint)

[The second constraint reflects the fact that it is not possible to manufacture a negative number of units.]

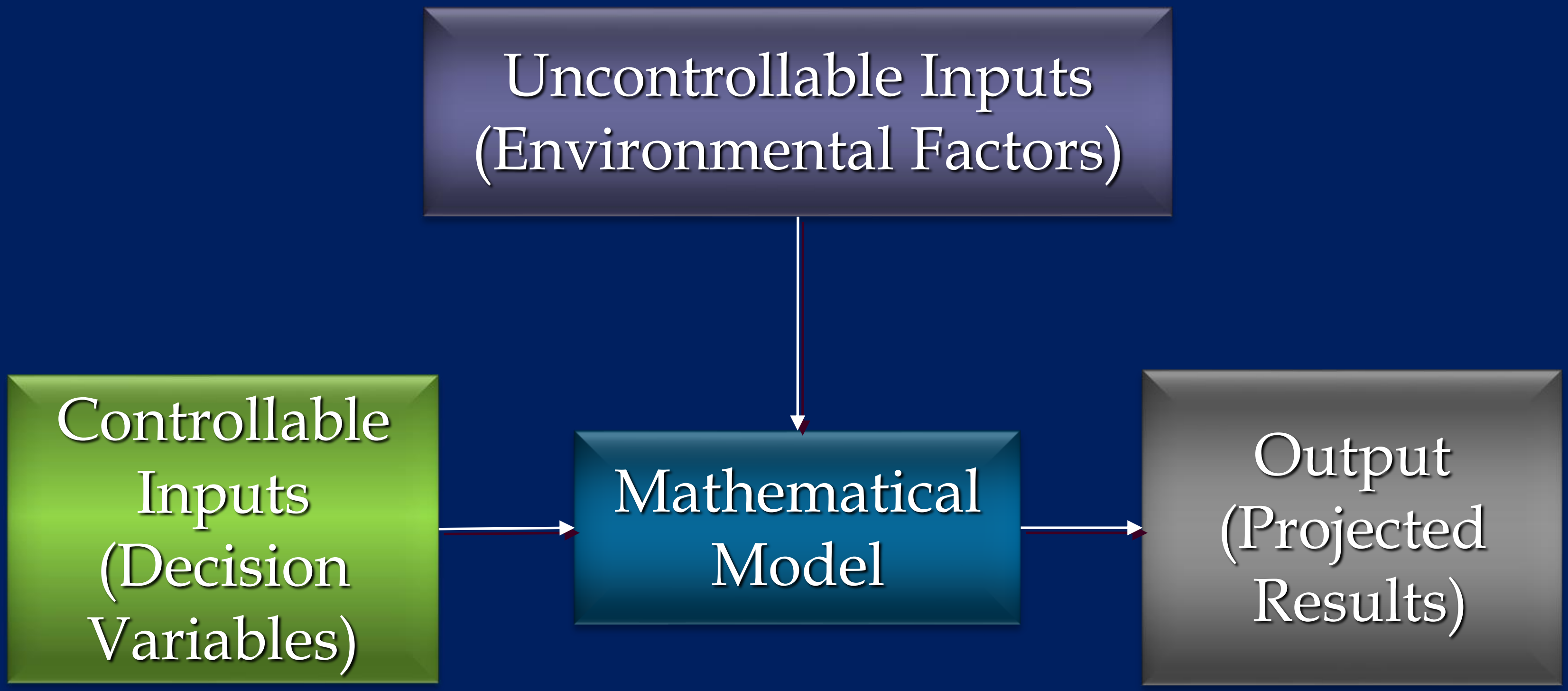
Mathematical Models

- Deterministic Model – if all uncontrollable inputs to the model are known and cannot vary
- Stochastic (or Probabilistic) Model – if any uncontrollable are uncertain and subject to variation
- Stochastic models are often more difficult to analyze.
    - In our simple production example, if the number of hours of production time per unit could vary from 3 to 6 hours depending on the quality of the raw material, the model would be stochastic.

Mathematical Models

- Cost/benefit considerations must be made in selecting an appropriate mathematical model.
- Frequently a less complicated (and perhaps less precise) model is more appropriate than a more complex and accurate one due to cost and ease of solution considerations.

Transforming Model Inputs into Output



Data Preparation

- Data preparation is not a trivial step, due to the time required and the possibility of data collection errors.
- A model with 50 decision variables and 25 constraints could have over 1300 data elements!
- Often, a fairly large data base is needed.
- Information systems specialists might be needed.

Model Solution

- The analyst attempts to identify the alternative (the set of decision variable values) that provides the "best" output for the model.
- The "best" output is the optimal solution.
- If the alternative does not satisfy all of the model constraints, it is rejected as being infeasible, regardless of the objective function value.
- If the alternative satisfies all of the model constraints, it is feasible and a candidate for the "best" solution.

Model Solution

- Trial-and-Error Solution for Production Problem

Production Quantity	Projected Profit	Total Hours of Production	Feasible Solution
0	0	0	Yes
2	20	10	Yes
4	40	20	Yes
6	60	30	Yes
8	80	40	Yes
10	100	50	No
12	120	60	No

Model Solution

- A variety of software packages are available for solving mathematical models.
  - *Microsoft Excel*
  - *LINGO*

Model Testing and Validation

- Often, goodness/accuracy of a model cannot be assessed until solutions are generated.
- Small test problems having known, or at least expected, solutions can be used for model testing and validation.
- If the model generates expected solutions, use the model on the full-scale problem.
- If inaccuracies or potential shortcomings inherent in the model are identified, take corrective action such as:
  - Collection of more-accurate input data
  - Modification of the model

Report Generation

- A managerial report, based on the results of the model, should be prepared.
- The report should be easily understood by the decision maker.
- The report should include:
  - the recommended decision
  - other pertinent information about the results (for example, how sensitive the model solution is to the assumptions and data used in the model)

Implementation and Follow-Up

- Successful implementation of model results is of critical importance.
- Secure as much user involvement as possible throughout the modeling process.
- Continue to monitor the contribution of the model.
- It might be necessary to refine or expand the model.

Models of Cost, Revenue, and Profit

Iron Works, Inc. manufactures two products made from steel and just received this month's allocation of b pounds of steel. It takes a_1 pounds of steel to make a unit of product 1 and a_2 pounds of steel to make a unit of product 2.

Let x_1 and x_2 denote this month's production level of product 1 and product 2, respectively. Denote by p_1 and p_2 the unit profits for products 1 and 2, respectively.

Iron Works has a contract calling for at least m units of product 1 this month. The firm's facilities are such that at most u units of product 2 may be produced monthly.

Example: Iron Works, Inc.

- Mathematical Model
  - The total monthly profit =

(profit per unit of product 1)

x (monthly production of product 1)

+ (profit per unit of product 2)

x (monthly production of product 2)

$$= p_1x_1 + p_2x_2$$

We want to maximize total monthly profit:

$$\text{Max} \quad p_1x_1 + p_2x_2$$

Example: Iron Works, Inc.

- Mathematical Model (continued)
  - The total amount of steel used during monthly production equals:

(steel required per unit of product 1)

x (monthly production of product 1)

+ (steel required per unit of product 2)

x (monthly production of product 2)

$$= a_1x_1 + a_2x_2$$

This quantity must be less than or equal to the allocated b pounds of steel:

$$a_1x_1 + a_2x_2 \leq b$$

Example: Iron Works, Inc.

- Mathematical Model (continued)
  - The monthly production level of product 1 must be greater than or equal to m :

$$x_1 \geq m$$

  - The monthly production level of product 2 must be less than or equal to u :

$$x_2 \leq u$$

  - However, the production level for product 2 cannot be negative:

$$x_2 \geq 0$$

Example: Iron Works, Inc.

- Mathematical Model Summary

$$
\begin{aligned}
\text{Max} \quad & p_1x_1 + p_2x_2 \\
\text{s.t.} \quad & a_1x_1 + a_2x_2 \le b \\
& x_1 \ge m \\
& x_2 \le u \\
& x_2 \ge 0
\end{aligned}
$$

Objective Function

Constraints

"Subject to"

Example: Iron Works, Inc.

- Question:

  Suppose $b = 2000$, $a_1 = 2$, $a_2 = 3$, $m = 60$, $u = 720$, $p_1 = 100$, and $p_2 = 200$. Rewrite the model with these specific values for the uncontrollable inputs.

Example: Iron Works, Inc.

- Answer:

  Substituting, the model is:

$$
\begin{array}{llcrcr}
\text{Max} & 100x_1 + 200x_2 & & & & \\
\text{s.t.} & 2x_1 & + & 3x_2 & \leq & 2000 \\
 & x_1 & & & \geq & 60 \\
 & & & x_2 & \leq & 720 \\
 & & & x_2 & \geq & 0
\end{array}
$$

Example: Iron Works, Inc.

- Question:

  The optimal solution to the current model is x_1 = 60 and x_2 = 626 2/3. If the product were engines, explain why this is not a true optimal solution for the "real-life" problem.

- Answer:

  One cannot produce and sell 2/3 of an engine. Thus the problem is further restricted by the fact that both x_1 and x_2 must be integers. (They could remain fractions if it is assumed these fractions are work in progress to be completed the next month.)

Example: Iron Works, Inc.

Uncontrollable Inputs

$100 profit per unit Prod. 1
$200 profit per unit Prod. 2
2 lbs. steel per unit Prod. 1
3 lbs. Steel per unit Prod. 2
2000 lbs. steel allocated
60 units minimum Prod. 1
720 units maximum Prod. 2
0 units minimum Prod. 2

Controllable Inputs

60 units Prod. 1
626.67 units Prod. 2

Mathematical Model

$$
\begin{aligned}
\text{Max} \quad & 100(60) + 200(626.67) \\
\text{s.t.} \quad & 2(60) + 3(626.67) \le 2000 \\
& 60 \ge 60 \\
& 626.67 \le 720 \\
& 626.67 \ge 0
\end{aligned}
$$

Output

Profit = $131,333.33
Steel Used = 2000

Example: Ponderosa Development Corp.

Ponderosa Development Corporation (PDC) is a small real estate developer that builds only one style cottage. The selling price of the cottage is $115,000.

Land for each cottage costs $55,000 and lumber, supplies, and other materials run another $28,000 per cottage. Total labor costs are approximately $20,000 per cottage.

Example: Ponderosa Development Corp.

Ponderosa leases office space for $2,000 per month. The cost of supplies, utilities, and leased equipment runs another $3,000 per month.

The one salesperson of PDC is paid a commission of $2,000 on the sale of each cottage. PDC has seven permanent office employees whose monthly salaries are given on the next slide.

Example: Ponderosa Development Corp.

Employee	Monthly Salary
President	$10,000
VP, Development	6,000
VP, Marketing	4,500
Project Manager	5,500
Controller	4,000
Office Manager	3,000
Receptionist	2,000

Example: Ponderosa Development Corp.

- Question:

  Identify all costs and denote the marginal cost and marginal revenue for each cottage.

- Answer:

  The monthly salaries total $35,000 and monthly office lease and supply costs total another $5,000. This $40,000 is a monthly fixed cost.

  The total cost of land, material, labor, and sales commission per cottage, $105,000, is the marginal cost for a cottage.

  The selling price of $115,000 is the marginal revenue per cottage.

Example: Ponderosa Development Corp.

- Question:

  Write the monthly cost function $c(x)$, revenue function $r(x)$, and profit function $p(x)$.

- Answer:

  $c(x)$ = variable cost + fixed cost = $105{,}000x + 40{,}000$

  $r(x) = 115{,}000x$

  $p(x) = r(x) - c(x) = 10{,}000x - 40{,}000$

Example: Ponderosa Development Corp.

- Question:

  What is the breakeven point for monthly sales of the cottages?

- Answer:

  $r(x) = c(x)$

  $115{,}000x = 105{,}000x + 40{,}000$

  Solving, $x = 4$ cottages.

Example: Ponderosa Development Corp.

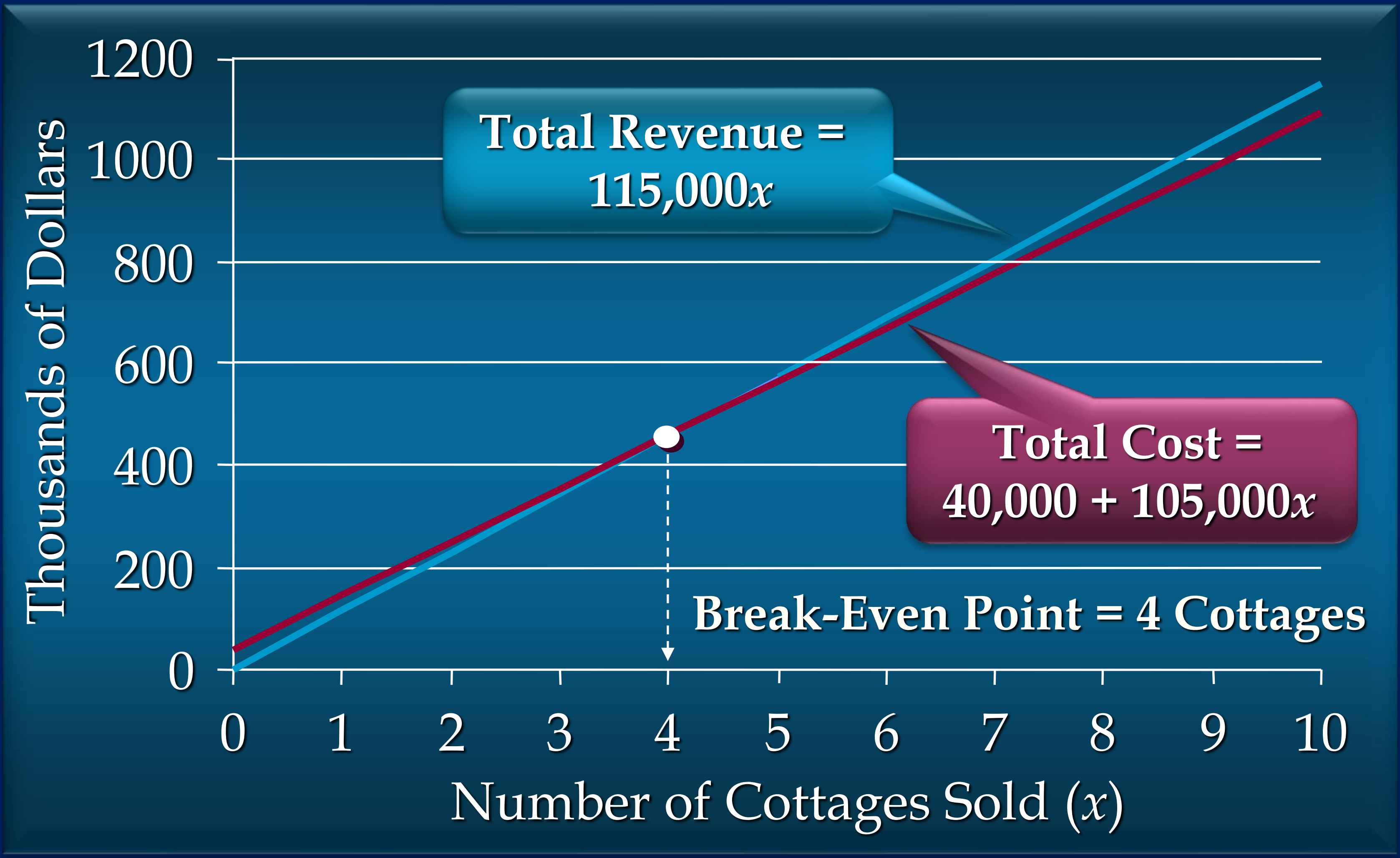
- Question:

  What is the monthly profit if 12 cottages per month are built and sold?

- Answer:

  $p(12) = 10{,}000(12) - 40{,}000 =$ **$80,000 monthly profit**

Example: Ponderosa Development Corp.



Using Excel for Breakeven Analysis

- A spreadsheet software package such as *Microsoft Excel* can be used to perform a quantitative analysis of Ponderosa Development Corporation.
- We will enter the problem data in the top portion of the spreadsheet.
- The bottom of the spreadsheet will be used for model development.

Example: Ponderosa Development Corp.

- Formula Spreadsheet

	A	B
1	PROBLEM DATA	
2	Fixed Cost	$40,000
3	Variable Cost Per Unit	$105,000
4	Selling Price Per Unit	$115,000
5	MODEL	
6	Sales Volume	
7	Total Revenue	=B4*B6
8	Total Cost	=B2+B3*B6
9	Total Profit (Loss)	=B7-B8

Example: Ponderosa Development Corp.

- Question

  What is the monthly profit if 12 cottages are built and sold per month?

Example: Ponderosa Development Corp.

- Spreadsheet Solution

	A	B
1	PROBLEM DATA	
2	Fixed Cost	$40,000
3	Variable Cost Per Unit	$105,000
4	Selling Price Per Unit	$115,000
5	MODEL	
6	Sales Volume	12
7	Total Revenue	$1,380,000
8	Total Cost	$1,300,000
9	Total Profit (Loss)	$80,000

Example: Ponderosa Development Corp.

- Question:

  What is the breakeven point for monthly sales of the cottages?

- Spreadsheet Solution:
  - One way to determine the break-even point using a spreadsheet is to use the Goal Seek tool.
  - *Microsoft Excel* ‘s Goal Seek tool allows the user to determine the value for an input cell that will cause the output cell to equal some specified value.
  - In our case, the goal is to set Total Profit to zero by seeking an appropriate value for Sales Volume.

Example: Ponderosa Development Corp.

- Spreadsheet Solution: Goal Seek Approach

Using *Excel*'s **Goal Seek** Tool

Step 1: Select **Data** tab at the top of the ribbon

Step 2: Select **What-If Analysis** in **Data Tools** group

Step 3: Select **Goal Seek** in **What-If Analysis**

Step 4: When the **Goal Seek** dialog box appears:

Enter B9 in the **Set cell** box

Enter 0 in the **To value** box

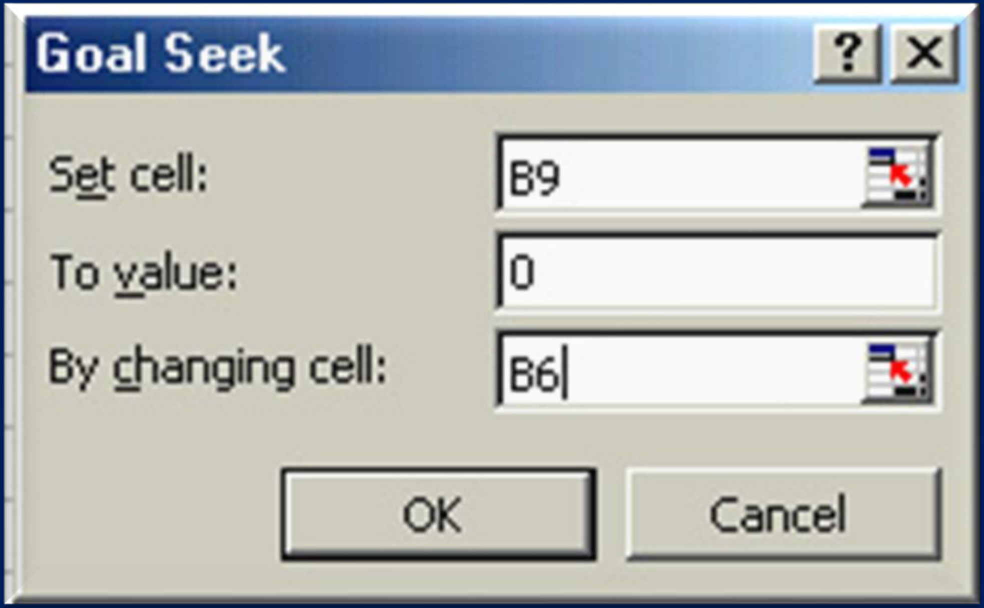
Enter B6 in the **By changing cell** box

Click **OK**

Example: Ponderosa Development Corp.

- Spreadsheet Solution: Goal Seek Approach

Completed Goal Seek Dialog Box



Example: Ponderosa Development Corp.

- Spreadsheet Solution: Goal Seek Approach

	A	B
1	PROBLEM DATA	
2	Fixed Cost	$40,000
3	Variable Cost Per Unit	$105,000
4	Selling Price Per Unit	$115,000
5	MODEL	
6	Sales Volume	4
7	Total Revenue	$460,000
8	Total Cost	$460,000
9	Total Profit (Loss)	$0

Quantitative Methods

- Linear programming is a problem-solving approach developed for situations involving maximizing or minimizing a linear function subject to linear constraints that limit the degree to which the objective can be pursued.
- Integer linear programming is an approach used for problems that can be set up as linear programs with the additional requirement that some or all of the decision recommendations be integer values.

Quantitative Methods

- Project scheduling: PERT (Program Evaluation and Review Technique) and CPM (Critical Path Method) help managers responsible for planning, scheduling, and controlling projects that consist of numerous separate jobs or tasks performed by a variety of departments, individuals, and so forth.
- Inventory models are used by managers faced with the dual problems of maintaining sufficient inventories to meet demand for goods and, at the same time, incurring the lowest possible inventory holding costs.

Quantitative Methods

- Waiting line (or queuing) models help managers understand and make better decisions concerning the operation of systems involving waiting lines.
- Simulation is a technique used to model the operation of a system. This technique employs a computer program to model the operation and perform simulation computations.

Quantitative Methods

- Decision analysis can be used to determine optimal strategies in situations involving several decision alternatives and an uncertain or risk-filled pattern of future events.
- Forecasting methods are techniques that can be used to predict future aspects of a business operation.
- Markov-process models are useful in studying the evolution of certain systems over repeated trials (such as describing the probability that a machine, functioning in one period, will function or break down in another period).