OPTIMIZATION TECHNIQUES

Linear Programming – II Graphical Method

Lecture 3 & 4

ITM BUSINESS SCHOOL
BUSINESS ANALYTICS
SEMESTER II
VIJAYANTA PAWASE

Linear Programming - II

Graphical Method

Introduction

- There are two methods available to find optimal solution to a Linear Programming Problem. One is graphical method and the other is simplex method.

- Graphical method can be used only for a two variables problem i.e., a problem which involves two decision variables. The two axes of the graph (X & Y axis) represent the two decision variables X_1 & X_2.

Methodology of Graphical Method

- **Step 1:** Formulation of LPP (Linear Programming Problem)
- **Step 2:** Determination of each axis.
- **Step 3:** Finding coordinates of constraint lines to represent constraint lines on the graph.
- **Step 4:** Representing constraint lines on the graph
- **Step 5:** Identification of Feasible Region
- **Step 6:** Finding the optimal solution

Methodology of Graphical Method

Step 1: Formulation of LPP (Linear Programming Problem)

- Use the given data to formulate the LPP.

Maximization:

Example 1: A company manufactures two products A and B. Both products are processed on two machines M_1 & M_2.

	M_1	M_2
A	6 Hrs./unit	2 Hrs/unit
B	4 Hrs./unit	4 Hrs./unit
Availability	7200 Hrs./month	4000 Hrs./month

Profit per unit for A is Rs. 100 and for B is Rs. 80. Find out monthly production of A and B to maximize profit by graphical method.

Methodology of Graphical Method

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Step 1: Formulation as LPP

X_1 = No. of units of A/month

X_2 = No. of units of B/month

Max. $Z = 100X_1 + 80X_2$

Subject to constraints:

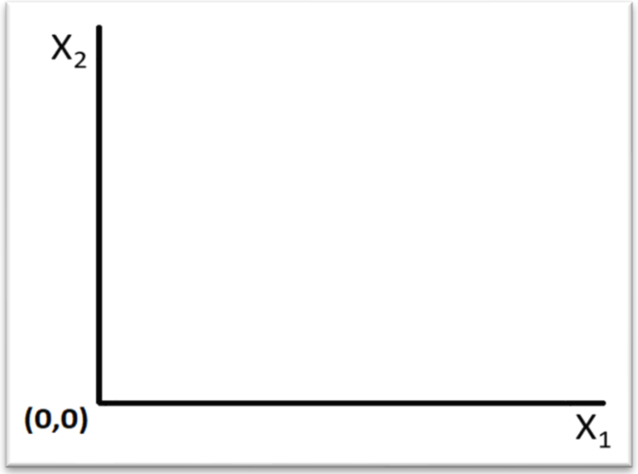
$$6X_1 + 4X_2 \leq 7200$$

$$2X_1 + 4X_2 \leq 4000$$

$$X_1, X_2 \geq 0$$

Step 2: Determination of each axis.

- Horizontal (X) axis: Product A (X_1)
- Vertical (Y) axis: Product B (X_2)



Methodology of Graphical Method

Step 3: Finding coordinates of constraint lines to represent constraint lines on the graph.

- The constraints are present in the form of inequality (≤). We should convert them into equality to obtain coordinates.

(1) Constraint No.1: $6X_1 + 4X_2 \leq 7200$

- Converting into equality:

$6X_1 + 4X_2 = 7200$

X_1 is the intercept on X axis and X_2 is the intercept on Y axis.

- To find X_1, let $X_2 = 0$

$6X_1 = 7200$ Therefore, $X_1, = 1200, X_2 = 0$

- To find X_2, let $X_1 = 0$

$4X_2 = 7200$ Therefore, $X_2 = 1800, X_1 = 0$

Hence the two points which make the constraint line are: **(1200,0) and (0,1800)**

Methodology of Graphical Method

Step 3: Finding coordinates of constraint lines to represent constraint lines on the graph.

- The constraints are present in the form of inequality (≤). We should convert them into equality to obtain coordinates.

(2) Constraint No.2: $2X_l + 4X_2 \leq 4000$

- To find X_1, let $X_2 = 0$

$2X_l = 4000$ Therefore, $X_1 = 2000$, $X_2 = 0$, (2000,0)

- To find X_2, let $X_1 = 0$

$4X_2 = 4000$ Therefore, $X_2 = 1000$, $X_1 = 0$, (0,1000)

Each constraint will be represented by a single straight line on the graph. There are two constraints, hence there will be two straight lines.

The co-ordinates of points are:

(1) Constraint No.1: (1200,0) and (0,1800)

(2) Constraint No.2: (2000,0) and (0,1000)

Methodology of Graphical Method

Step 4: Representing constraint lines on the graph

- To mark the points on the graph, we need to select the appropriate scale.

Which scale to take will depend on maximum value of X_1 & X_2 from co-ordinates.

- For X_1, we have 2 values → 1200 and 2000.

Therefore, Max. value for X_2 = 2000

- For X_2, we have 2 values → 1800 and 1000

Therefore, Max. value for X_2 = 1800

Assuming that we have a graph paper 20 x 30 cm. We need to accommodate our lines such that for X axis, maximum value of 2000 contains in 20 cm.

Scale: 1 cm = 200 units
2000 units = 10 cm (X axis)
1800 units = 9cm (Y axis)
The scale should be such that the diagram should not be too small.

Methodology of Graphical Method

Step 4: Representing constraint lines on the graph

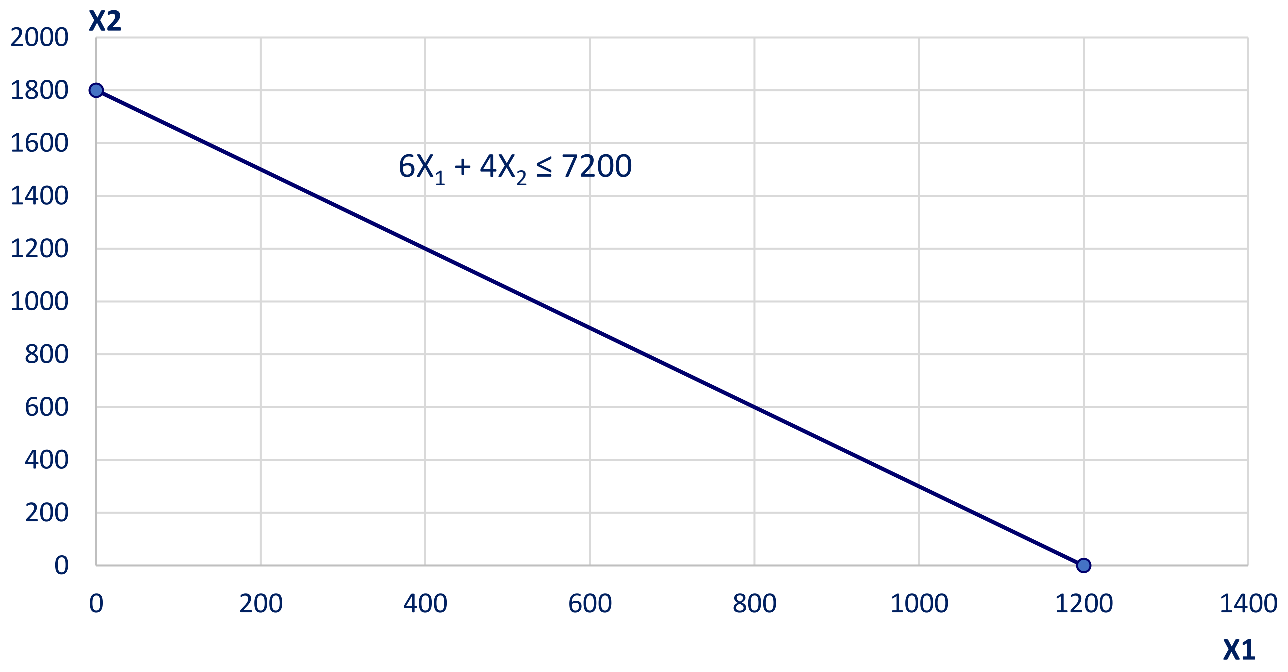
- To mark the points on the graph, we need to select the appropriate scale.

Constraint No.1:

The line joining the two points (1200,0) and (0,1800) represents the constraint $6X_1 + 4X_2 \leq 7200$.

Every point on the line will satisfy the equation (equality) $6X_1 + 4X_2 = 7200$.

Every point below the line will satisfy the inequality (less than) $6X_1 + 4X_2 < 7200$.



Methodology of Graphical Method

Step 4: Representing constraint lines on the graph

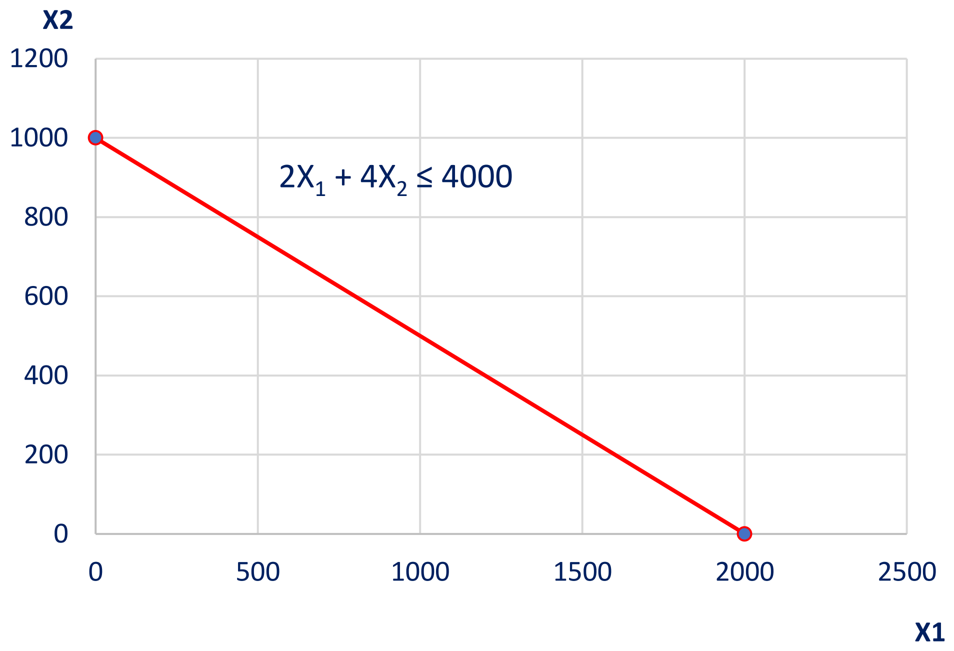
- To mark the points on the graph, we need to select the appropriate scale.

Constraint No.2:

The line joining the two points (2000,0) and (0,1000) represents the constraint $2X_1 + 4X_2 \leq 4000$.

Every point on the line will satisfy the equation (equality) $2X_1 + 4X_2 = 4000$.

Every point below the line will satisfy the inequality (less than) $2X_1 + 4X_2 < 4000$.

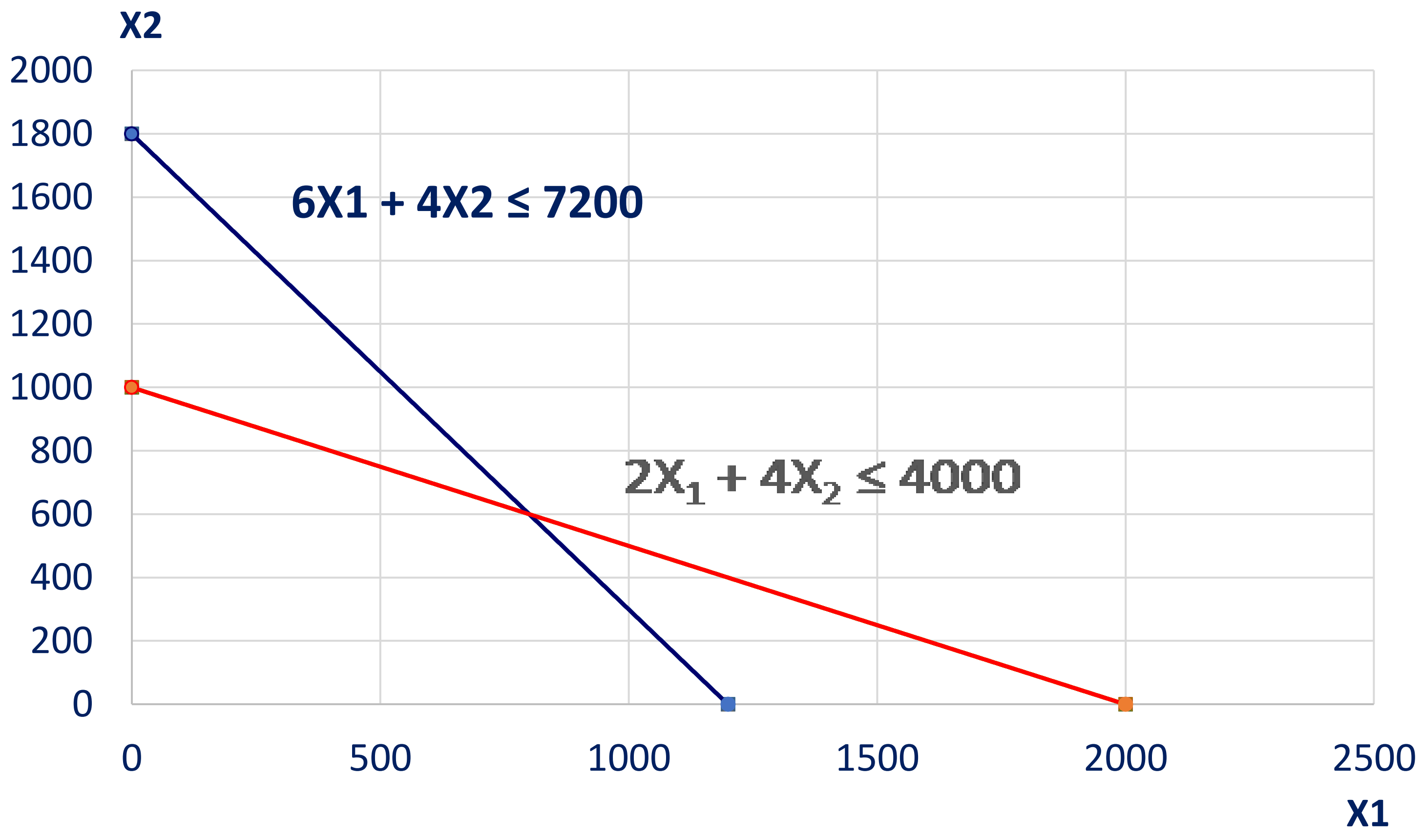


Methodology of Graphical Method

Step 4: Representing constraint lines on the graph

- To mark the points on the graph, we need to select the appropriate scale.

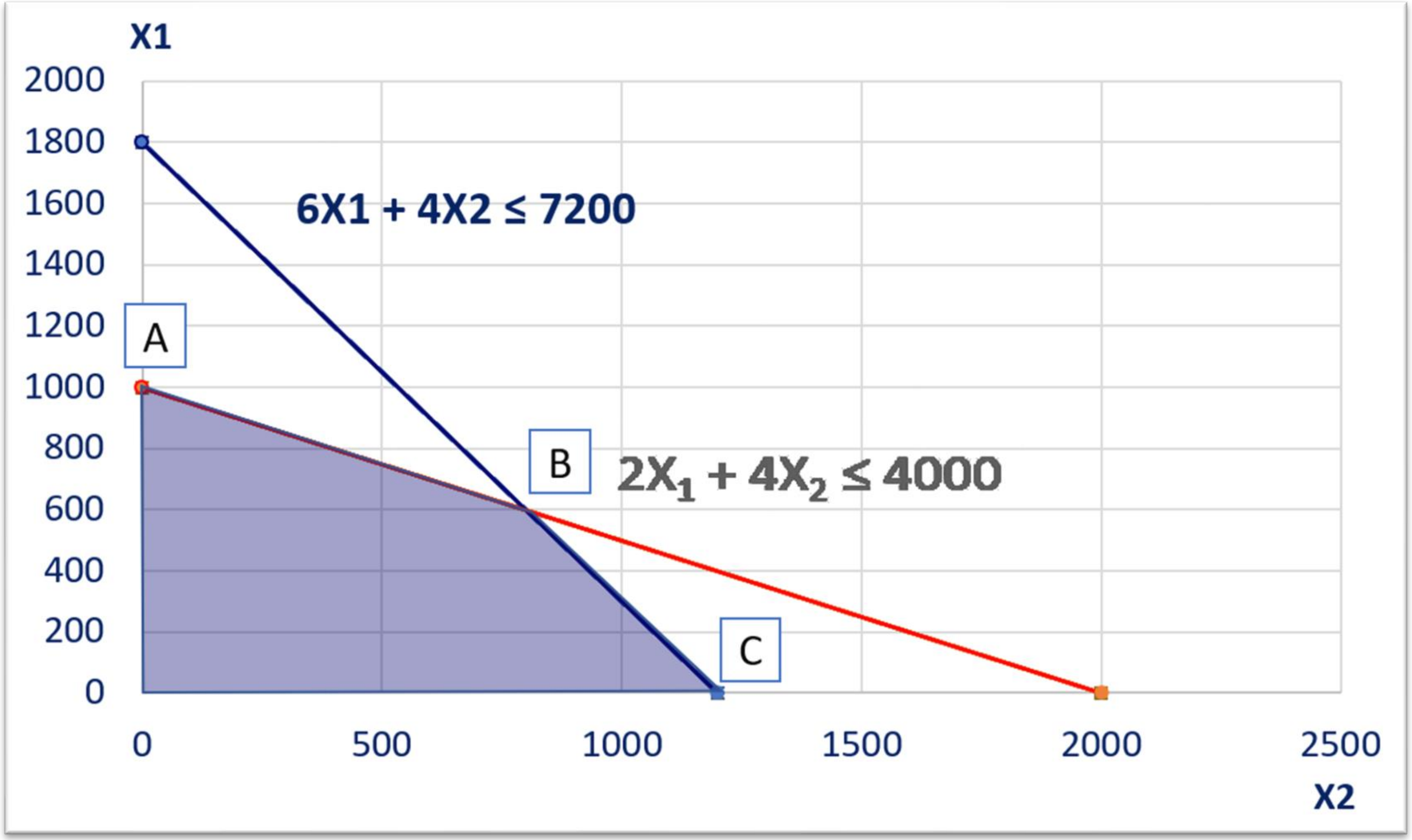
Now the graph will look like this:



Methodology of Graphical Method

Step 5: Identification of Feasible Region

- The feasible region is the region bounded by constraint lines. All points inside the feasible region or on the boundary of the feasible region or at the comers of the feasible region satisfy all constraints.



Both the constraints are 'less than or equal to' (≤) type. Hence, the feasible region should be **inside both constraint lines.**

Hence, the feasible region is the polygon OABC. '0' is the origin whose co-ordinates are (0,0). O, A, B and C are called vertices of the feasible region.

Methodology of Graphical Method

What is meant by the feasible region of a solution in the LPP graphical method?

- The feasible region of the solution means that part of the graphical solution which satisfies all the constraints of the problem. There may be different types of constraints in the problem e.g., ≥, ≤ etc. The feasible region is the common region to all constraints.
- To find the feasible region, we should eliminate all those parts of the solution which are not common to all constraints. The remaining region is the one that is the feasible region of solution.

Type of constraint	Location of feasible region
≤	Inside the constraint line
≥	Outside the constraint line
=	On the constraint line

Methodology of Graphical Method

Step 6: Finding the optimal solution

- The optimal 'solution always lies at one of the vertices or corners of the feasible region.
- To find optimal solution:
- We use corner point method. We find co-ordinates (X1 and X2 values) for each vertex or corner point. From this we find 'Z' value for each corner point.

Vertex	Co-ordinates	$Z = 100X_1 + 80X_2$
0	$X_1 = 0; X_2 = 0$	Z =0
	From Graph	
A	$X_1 = 0, X_2 = 1000$	Z = Rs. 80,000
	From Graph	
B	$X_1 = 800, X_2 = 600$	Z = Rs.128,000
	From simultaneous equations	
C	$X_1 = 1200, X_2 = 0$	Z = Rs. 120,000
	From Graph	

For B → B is at the intersection of two constraint lines $6X_l + 4X_2 = 7200$ and 2X1 + 4X2 = 4000. Hence, values of X1 and X_2 at B must satisfy both the equations.

Max. Z = Rs. 128,000 (At point B)

Solution:
Optimal profit = Max. Z = Rs. 128,000
Product Mix:
X_1 = No. of units of A/month = 800
X_2 = No. of units of B/month = 600

Methodology of Graphical Method

Step 1: Formulation of LPP (Linear Programming Problem)

- Use the given data to formulate the LPP.

Minimization:

Example 2: A firm is engaged in animal breeding. The animals are to be given nutrition supplements every day. There are two products A and B which contain the three required nutrients.

Nutrients	Quantity/unit		Minimum requirement
	A	**B**	
1	72	12	216
2	6	24	72
3	40	20	200

Product cost per unit is - A: Rs. 40, B: Rs. 80. Find out the quantity of product A & B to be given to provide minimum nutritional requirement.

Methodology of Graphical Method

Minimization:

Example 2: A firm is engaged in animal breeding. The animals are to be given nutrition supplements every day. There are two products A and B which contain the three required nutrients.

Nutrients	Quantity/unit		Minimum requirement
	A	B	
1	72	12	216
2	6	24	72
3	40	20	200

Product cost per unit is - A: Rs. 40, B: Rs. 80. Find out the quantity of product A & B to be given to provide minimum nutritional requirement.

Step 1: Formulation as LPP

X_1 = No. of units of A

X_2 = No. of units of B

Z = Total cost

$$\textbf{Min. } Z = 40X_1 + 80X_2$$

Subject to constraints:

$72X_1 + 12X_2 \geq 216$

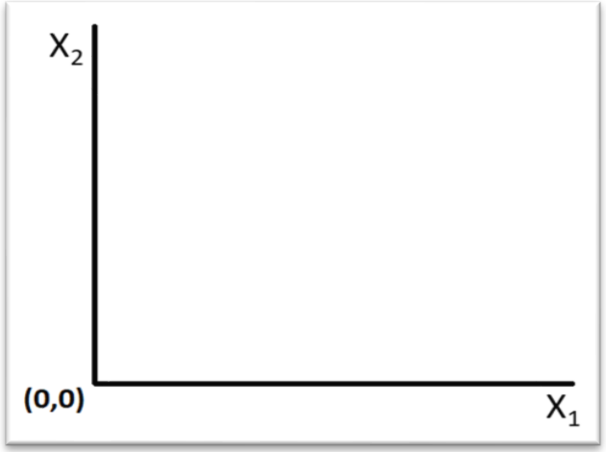
$6X_1 + 24X_2 \geq 72$

$40X_1 + 20X_2 \geq 200$

$X_1, X_2 \geq 0$

Step 2: Determination of each axis.

- Horizontal (X) axis: Product A (X_1)
- Vertical (Y) axis: Product B (X_2)



Methodology of Graphical Method

Step 3: Finding coordinates of constraint lines to represent constraint lines on the graph.

- All constraints are 'greater than or equal to' type. We should convert them into equality.

(1) Constraint No.1: $72X_1 + 12X_2 \geq 216$

- Converting into equality:

$72X_1 + 12X_2 = 216$

X_1 is the intercept on X axis and X_2 is the intercept on Y axis.

- To find X_1, let $X_2 = 0$

$72X_1 = 216$ Therefore, $X_1, = 3$, $X_2 = 0$

- To find X_2, let $X_1 = 0$

$12X_2 = 216$ Therefore, $X_2 = 18$, $X_1 = 0$

Hence the two points which make the constraint line are: **(3,0) and (0,18)**

Methodology of Graphical Method

Step 3: Finding coordinates of constraint lines to represent constraint lines on the graph.

- All constraints are 'greater than or equal to' type. We should convert them into equality.

(2) Constraint No.2: $6X_1 + 24X_2 \geq 72$

- Converting into equality:

$6X_1 + 24X_2 = 72$

X_1 is the intercept on X axis and X_2 is the intercept on Y axis.

- To find X_1, let $X_2 = 0$

$6X_1 = 72$ Therefore, X_1, $= 12$, $X_2 = 0$

- To find X_2, let $X_1 = 0$

$24X_2 = 72$ Therefore, $X_2 = 3$, $X_1 = 0$

Hence the two points which make the constraint line are: **(12,0) and (0,3)**

Methodology of Graphical Method

Step 3: Finding coordinates of constraint lines to represent constraint lines on the graph.

- All constraints are 'greater than or equal to' type. We should convert them into equality.

(3) Constraint No.3: $40X_1 + 20X_2 \geq 200$

- Converting into equality:

$40X_1 + 20X_2 = 200$

X_1 is the intercept on X axis and X_2 is the intercept on Y axis.

- To find X_1, let $X_2 = 0$

$40X_1 = 200$ Therefore, $X_1, = 5$, $X_2 = 0$

- To find X_2, let $X_1 = 0$

$20X_2 = 200$ Therefore, $X_2 = 10$, $X_1 = 0$

Hence the two points which make the constraint line are: **(5,0) and (0,3)**

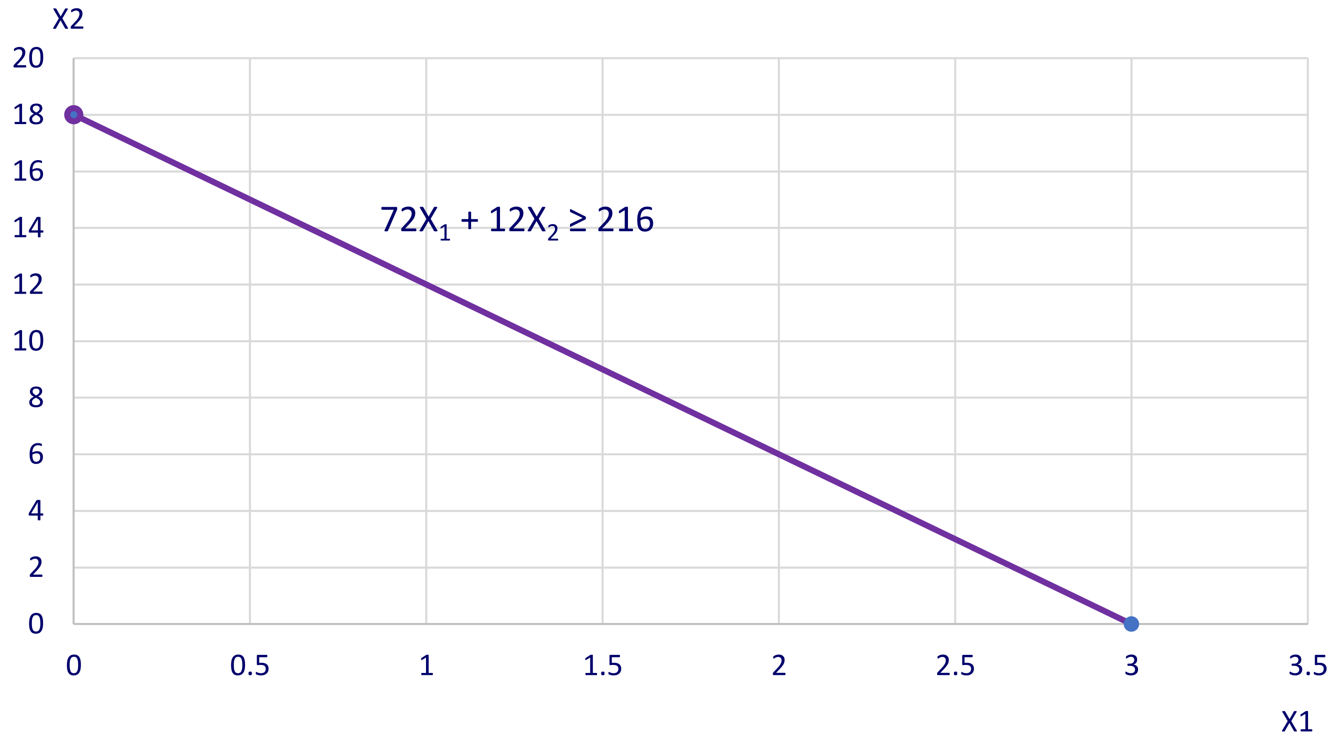
Methodology of Graphical Method

Step 4: Representing constraint lines on the graph

- To mark the points on the graph, we need to select the appropriate scale.

To select scale:
Maximum value for X_1 = 12
Maximum value for X_2 = 18
Scale: 1 cm = 1 unit

Constraint No.1:



Every point **on the line** will satisfy the equation (equality) $72X_1 + 12X_2 = 216$.

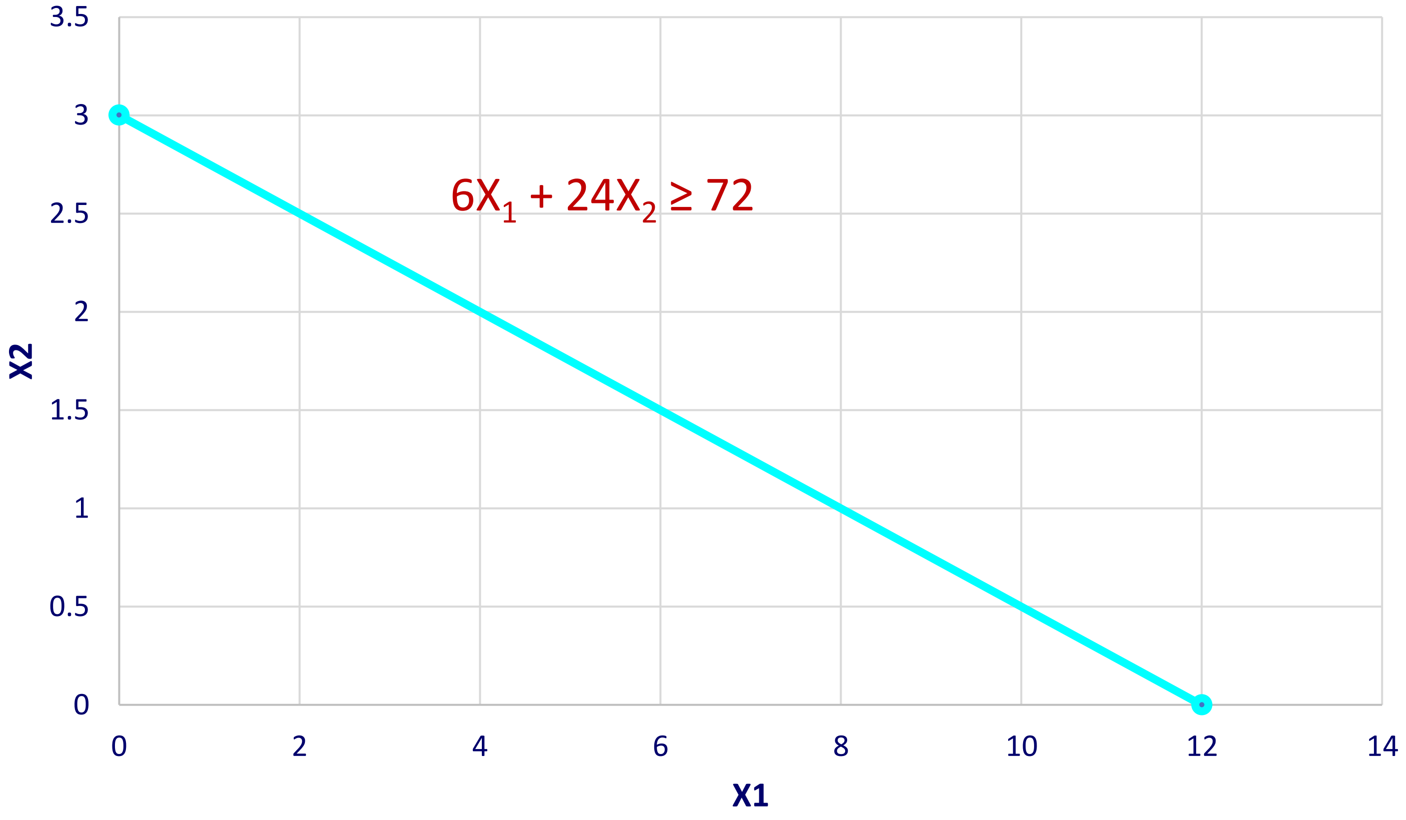
Every point **above the line** will satisfy the inequality (greater than) $72X_1 + 12X_2 > 216$.

Methodology of Graphical Method

Step 4: Representing constraint lines on the graph

- To mark the points on the graph, we need to select the appropriate scale.

Constraint No.2:

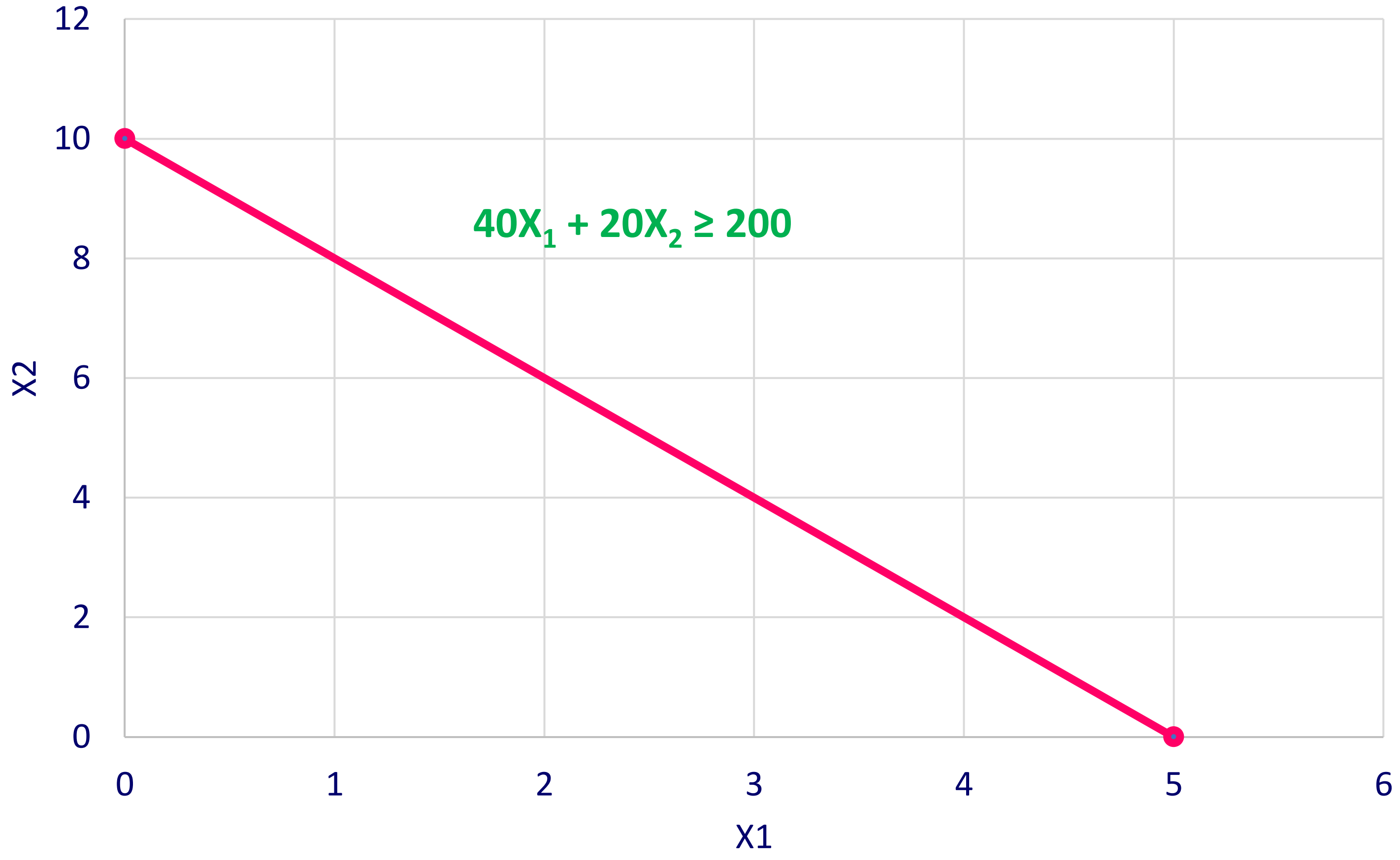


Methodology of Graphical Method

Step 4: Representing constraint lines on the graph

- To mark the points on the graph, we need to select the appropriate scale.

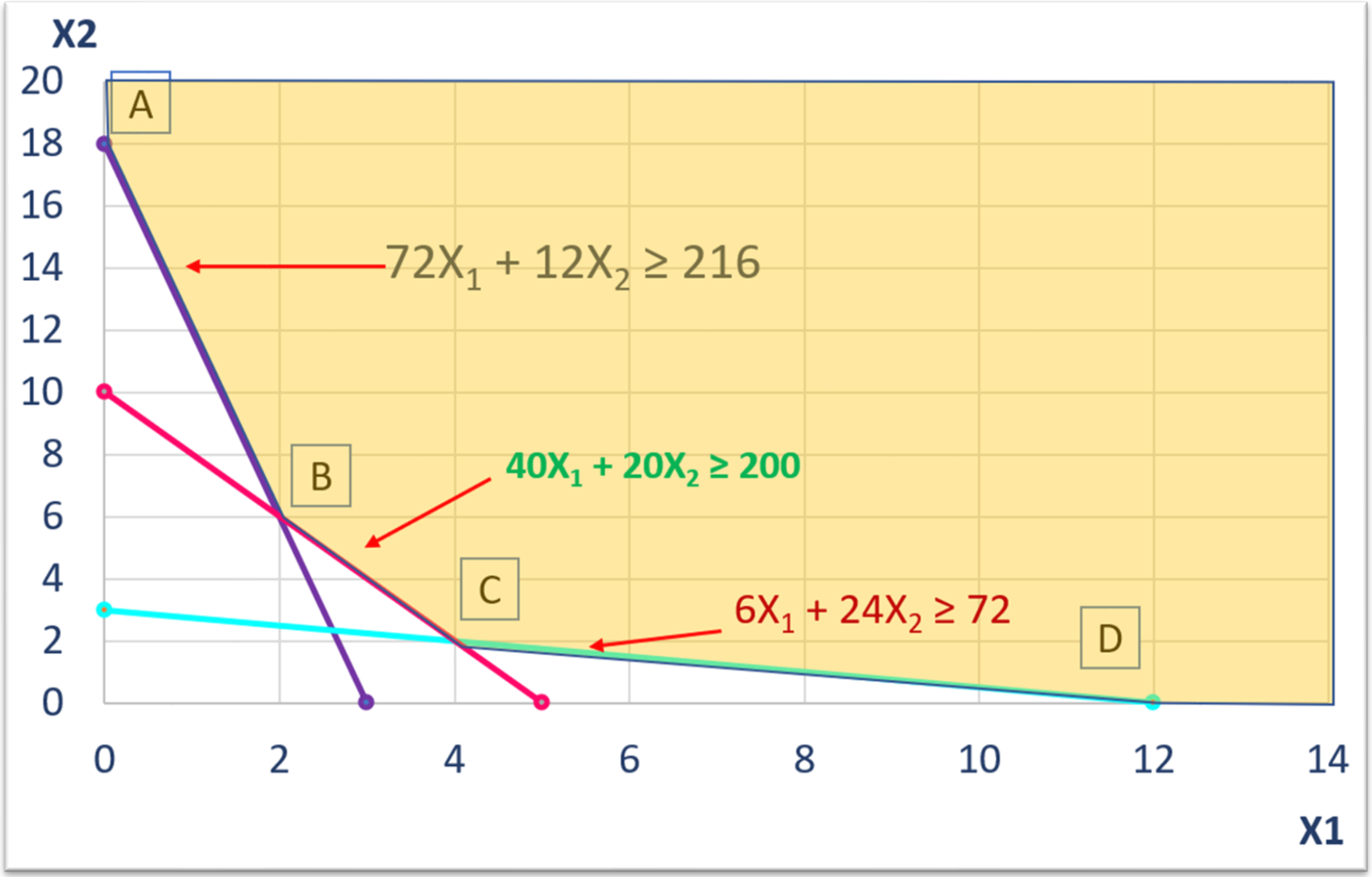
Constraint No.3:



Methodology of Graphical Method

Step 5: Identification of Feasible Region

- The final graph will look like:



All constraints are greater than or equal to (≥) type. Hence, feasible region should be above (to the right of) all constraints. The vertices of the feasible region are A,B,C & D.

Methodology of Graphical Method

Step 6: Finding the optimal solution

- The optimal 'solution always lies at one of the vertices or corners of the feasible region.
- To find optimal solution:
- We use corner point method. We find co-ordinates (X1 and X2 values) for each vertex or corner point. From this we find 'Z' value for each corner point.

Corner point method:

Vertex	**Co-ordinates**	**$Z = 40X_1 + 80X_2$**
A	$X_1 = 0; X_2 = 18$	Z =1440
	From Graph	
B	$X_1 = 2, X_2 = 6$	Z = Rs. 560
	From simultaneous equations	
C	$X_1 = 4, X_2 = 2$	Z = Rs.320
	From simultaneous equations	
D	$X_1 = 12, X_2 = 0$	Z = Rs. 480
	From Graph	

Therefore, Min Z = Rs.320 (At point 'C')

Solution:

Optimal cost = Min Z = Rs. 320

Optimal Product Mix:

X_1 = No. of units of product A= 4

X_2 = No. of units of product B = 2

Methodology of Graphical Method

Step 1: Formulation of LPP (Linear Programming Problem)

- Use the given data to formulate the LPP.

Maximization-Mixed Constraints:

Example 3: A firm makes two products P_1 & P_2 and has a production capacity of 18 tonnes per day. P_1 & P_2 require the same production capacity. The firm must supply at least 4 t of P_1 & 6 t of P_2 per day. Each tonne of P_1 & P_2 requires 60 hrs of machine work each. The maximum number of machine-hours available is 720. Profit per tonne for P_1 is Rs.160 and for P_2 is Rs. 240. Find the optimal solution by the graphical method.

Methodology of Graphical Method

Maximization-Mixed Constraints:

Example 3: A firm makes two products P_1 & P_2 and has a production capacity of 18 tonnes per day. P_1 & P_2 require the same production capacity. The firm must supply at least 4 t of P_1 & 6 t of P_2 per day. Each tonne of P_1 & P_2 requires 60 hrs of machine work each. The maximum number of machine-hours available is 720. Profit per tonne for P_1 is Rs.160 and for P_2 is Rs. 240. Find the optimal solution by the graphical method.

Step 1: Formulation as LPP

X_1 = Tonnes of P_1/Day

X_2 = Tonnes of P_2/Day

Max. $Z = 160X_1 + 240X_2$

Subject to constraints:

$$X_1 \geq 4$$

$$X_2 \geq 6$$

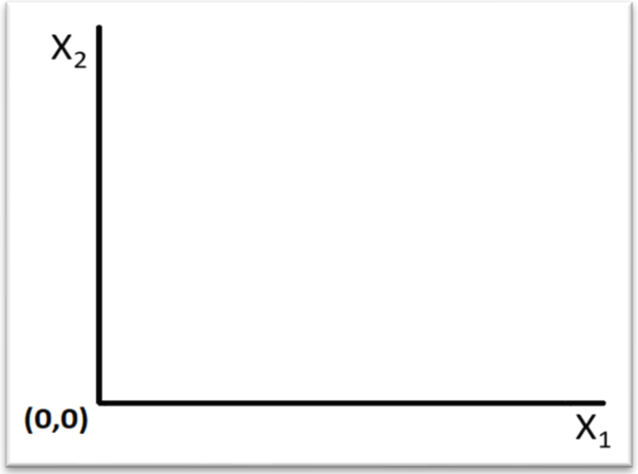
$$X_1 + X_2 \leq 18$$

$$60X_1 + 60X_2 \leq 720$$

$$X_1, X_2 \geq 0$$

Step 2: Determination of each axis.

- Horizontal (X) axis: Product P_1 (X_1)
- Vertical (Y) axis: Product P_2 (X_2)



Methodology of Graphical Method

Step 3: Finding coordinates of constraint lines to represent constraint lines on the graph.

- The constraints are present in the form of inequality (≤ or ≥). We should convert them into equality to obtain coordinates.

Coordinates for constraint lines :

(1) $X_1 \geq 4$ _ _ _ _ _ (4,0) _ _ _ _ _ No, value for X_2, Therefore, $X_2 = 0$

(2) $X_2 \geq 6$ _ _ _ _ _ (0,6) _ _ _ _ _ No, value for X_1, Therefore, $X_1 = 0$

(3) $X_1 + X_2 \leq 18$ _ _ _ _ _ (18,0), (0,18)

(4) $60X_1 + 60X_2 \leq 720$ _ _ _ _ _ (12,0), (0,12)

If, $X_1 = 0$, $60X_2 = 720$ _ _ _ _ Therefore, $X_2 = 12$ (0,12)

If, $X_2 = 0$, $60X_1 = 720$ _ _ _ _ Therefore, $X_1 = 12$ (12,0)

Methodology of Graphical Method

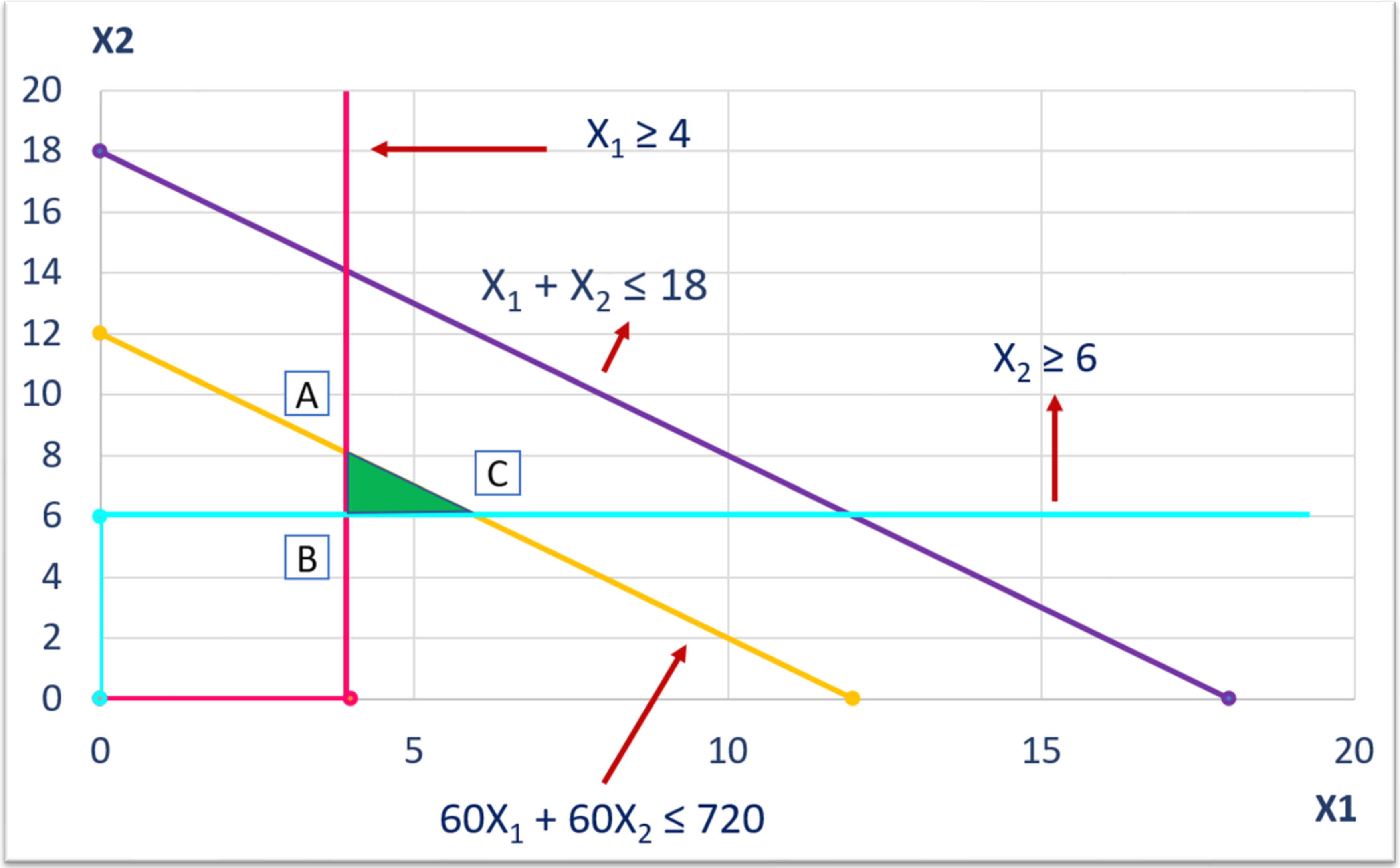
Step 4: Representing constraint lines on the graph

- To mark the points on the graph, we need to select the appropriate scale.

To select scale:
Maximum value for $X_1 = 18$
Maximum value for $X_2 = 18$
Scale: 1 cm = 2 Tonnes

Feasible Region: ABC

Two constraints are 'greater than or equal to' type. Hence, the feasible region will be above or to the right of these constraint lines. Two constraints are 'less than or equal to' type. Hence, the feasible region will be below or to the left of these constraint lines. Hence, the feasible region is ABC.



Methodology of Graphical Method

Step 6: Finding the optimal solution

- The optimal 'solution always lies at one of the vertices or corners of the feasible region.

Corner point method:		
Vertex	**Co-ordinates**	**$Z = 160X_1 + 240X_2$**
A	$X_1 = 4; X_2 = 8$	Z =Rs.2560
	From simultaneous equations	
B	$X_1 = 4, X_2 = 6$	Z = Rs. 2080
	From Graph	
C	$X_1 = 6, X_2 = 6$	Z = Rs.2400
	From simultaneous equations	

For A→X_1 = 4 from graph

A is on the line $60X_1 + 60X_2 = 720$

$60X_2 = 720 - 60\ (4) = 480$ _ _ _ _ Therefore, $X_2 = 8$

For C → X_2 = 6 from graph

C is on the line $60X_1 + 60X_2 = 720$

$60X_1 = 720 - 60\ (6) = 360$ _ _ _ _ Therefore, $X_1 = 6$

Z max = Rs. 2,560 [At point 'A']

Max. Z = Rs. 2560 (At point A)

Solution:

Optimal profit = Max Z = Rs. 2560

Optimal Product Mix:

X_1 = Production of P_1 = 4 Tonnes

X_2 = Tonnes of P_2 = 8 Tonnes

Methodology of Graphical Method

Step 1: Formulation of LPP (Linear Programming Problem)

- Use the given data to formulate the LPP.

Minimization-Mixed Constraints:

Example 4: A firm produces two products P and Q. Daily production upper limit is 600 units for total production. But at least 300 total units must be produced every day. Machine hours consumption per unit is 6 for P and 2 for Q. At least 1200 machine hours must be used daily. Manufacturing costs per unit are Rs. 50 for P and Rs. 20 for Q. Find the optimal solution for the LPP graphically.

Methodology of Graphical Method

Minimization-Mixed Constraints:

Example 4: A firm produces two products P and Q. Daily production upper limit is 600 units for total production. But at least 300 total units must be produced every day. Machine hours consumption per unit is 6 for P and 2 for Q. At least 1200 machine hours must be used daily. Manufacturing costs per unit are Rs. 50 for P and Rs. 20 for Q. Find the optimal solution for the LPP graphically.

Step 1: Formulation as LPP

X_1 = No. of Units of P per day

X_2 = No. of Units of Q per day

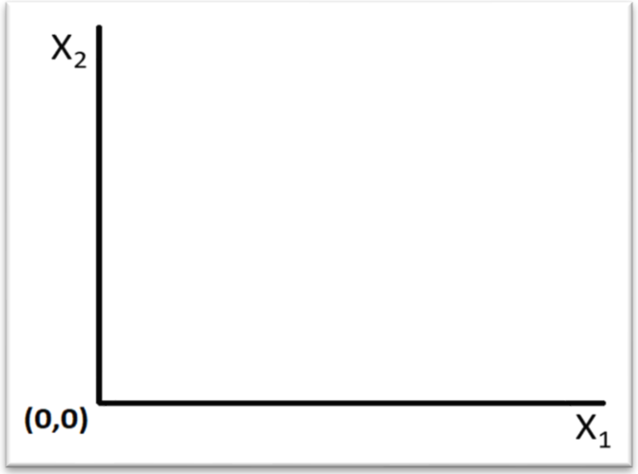
Min. Z = $50X_1 + 20X_2$

Subject to constraints:

$$X_1 + X_2 \leq 600$$
$$X_1 + X_2 \geq 300$$
$$6X_1 + 2X_2 \geq 1200$$
$$X_1, X_2 \geq 0$$

Step 2: Determination of each axis.

- Horizontal (X) axis: Product P (X_1)
- Vertical (Y) axis: Product Q (X_2)



Methodology of Graphical Method

Step 3: Finding coordinates of constraint lines to represent constraint lines on the graph.

- The constraints are present in the form of inequality (≤ or ≥). We should convert them into equality to obtain coordinates.

Coordinates for constraint lines :

(1) $X_1 + X_2 = 600$ ……. Converting in equality

If $X_1 = 0$, $X_2 = 600$, _ _ _ _ _ (0,600)

If $X_2 = 0$, $X_1 = 600$, _ _ _ _ _ (600,0)

(2) $X_1 + X_2 = 300$ ……. Converting in equality

If $X_1 = 0$, $X_2 = 300$, _ _ _ _ _ (0,300)

If $X_2 = 0$, $X_1 = 300$, _ _ _ _ _ (300,0)

(3) $6X_1 + 2X_2 = 1200$……. Converting in equality

If $X_1 = 0$, $2X_2 = 1200$, _ _ _ _ _ $X_2 = 600$…….. (0,600)

If $X_2 = 0$, $6X_1 = 1200$, _ _ _ _ _ $X_1 = 200$……..(200,0)

Methodology of Graphical Method

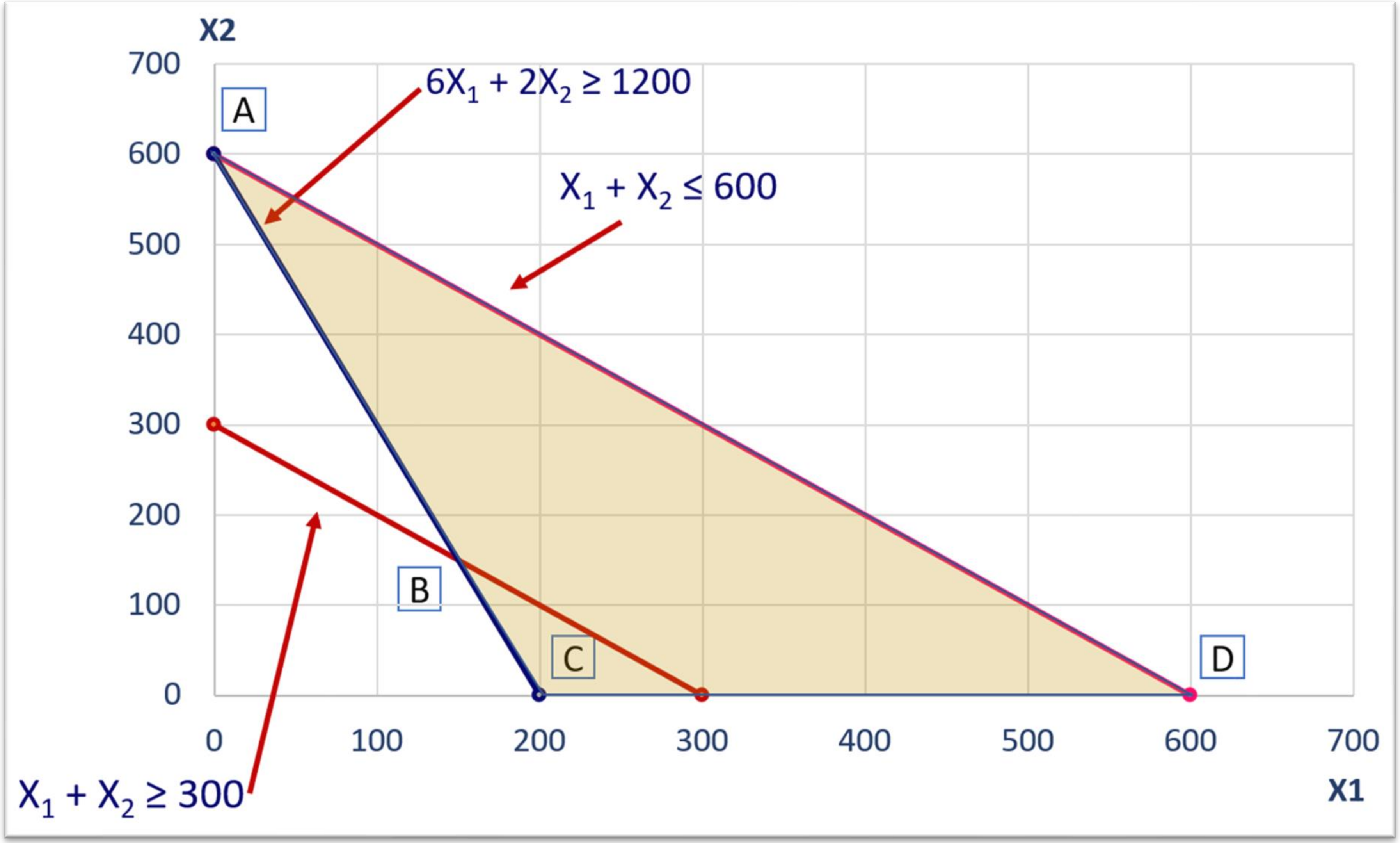
Step 4: Representing constraint lines on the graph

- To mark the points on the graph, we need to select the appropriate scale.

To select scale:
Maximum value for $X_1 = 600$
Maximum value for $X_2 = 600$
Scale: 1 cm = 50 units

Feasible Region: ABCD

Two constraints are of 'greater than or equal to' type. Hence, the feasible region is to the right of or above these constraint lines. One constraint is of 'less than or equal to' type. Hence, the feasible region is to the left of or below that constraint line.



Methodology of Graphical Method

Step 6: Finding the optimal solution

- The optimal 'solution always lies at one of the vertices or corners of the feasible region.

Corner point method:		
Vertex	**Co-ordinates**	**$Z = 50X_1 + 20X_2$**
A	$X_1 = 0; X_2 = 600$	Z =Rs.12,000
	From Graph	
B	$X_1 = 150, X_2 = 150$	Z = Rs. 10,500
	From simultaneous equations	
C	$X_1 = 300, X_2 = 0$	Z = Rs.15,000
	From simultaneous equations	
D	$X_1 = 600; X_2 = 0$	Z =Rs.30,000
	From Graph	

For B→B is at intersection of two constraint lines

$6X_1 + 2X_2 = 1200$ and $X_1 + X_2 = 300$

$6X_1 + 2X_2 = 1200$ ……(1)

$X_1 + X_2 = 300$ …..(2)

$2X_1 + 2X_2 = 600$ …..(2) x 2

Therefore, $4X_1 = 600$

Therefore, $X_1 = 150$

Substituting value in equation (2), $X_2 = 150$

Min. Z = Rs. 10,500 (At point B)

Solution:

Optimal cost = Min Z = Rs. 10,500

X_1 = No. of Units of P per day = 150

X_2 = No. of Units of Q per day = 150

Methodology of Graphical Method

*LINEAR PROGRAMMING FOR TRAFFIC CONTROL AT HANSHIN EXPRESSWAY**

The Hanshin Expressway was the first urban toll expressway in Osaka, Japan. Although in 1964 its length was only 2.3 kilometers, today it is a large-scale urban expressway network of 200 kilometers. The Hanshin Expressway provides service for the Hanshin (Osaka-Kobe) area, the second-most populated area in Japan. An average of 828,000 vehicles use the expressway each day, with daily traffic sometimes exceeding 1 million vehicles. In 1990 the Hanshin Expressway Public Corporation started using an automated traffic control system in order to maximize the number of vehicles flowing into the expressway network.

The automated traffic control system relies on two control methods: (1) limiting the number of cars that enter the expressway at each entrance ramp; and (2) providing drivers with up-to-date and accurate traffic information, including expected travel times and information about accidents. The approach used to limit the number of vehicles depends upon whether the expressway is in a normal or steady state of operation, or whether some type of unusual event, such as an accident or a breakdown, has occurred.

In the first phase of the steady-state case, the Hanshin system uses a linear programming model to maximize the total number of vehicles entering the system, while preventing traffic congestion and adverse effects on surrounding road networks. The data that drive the linear programming model are collected from detectors installed every 500 meters along the expressway and at all entrance and exit ramps. Every five minutes the real-time data collected from the detectors are used to update the model coefficients, and a new linear program computes the maximum number of vehicles the expressway can accommodate.

The automated traffic control system proved successful. According to surveys, traffic control decreased the length of congested portions of the expressway by 30% and the duration by 20%. In addition to its extreme cost-effectiveness, drivers consider it an indispensable service.

*Based on T. Yoshino, T. Sasaki, and T. Hasegawa, "The Traffic-Control System on the Hanshin Expressway," *Interfaces* (January/February 1995): 94–108.

Methodology of Graphical Method

Example 5: Print Well Pvt. Ltd is facing a tight financial squeeze and hence are attempting cost-saving wherever possible. The current contract is to print a book in hardcover and paperback. The cost of a hardcover type is Rs. 600 per 100 copies and Rs. 500 per 100 copies of paperback type. The company decides to run their two printing presses PI and PII for at least 80 hours and 60 hours respectively every week. PI can produce 100 hardcover books in 2 hours and 100 paperbacks in 1 hour. PII can produce 100 hardcover books in 1 hour and 100 paperbacks in 2 hours. Determine how many books of each type should be produced to minimize costs. Use the graphical method of linear programming.

Methodology of Graphical Method

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Step 1: Formulation as LPP

Decision Variables:

X_1 = No. of hardcover copies (in a lot of 100)

X_2 = No. of paperback copies (in a lot of 100)

Min. Z = $600X_1 + 500X_2$

Subject to constraints:?

Methodology of Graphical Method

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Constraints:

	Hardcover (in 100)	**Paperback (in 100)**	
PI (per 100)	2 hrs	1 hrs	At least 80 hrs.
PII (per 100)	1 hrs	2 hrs	At least 60 hrs.

Subject to constraints:

$$2X_1 + 1X_2 \geq 80 \text{ ....PI}$$

$$1X_1 + 2X_2 \geq 60 \text{ ....PII}$$

$$X_1, X_2 \geq 0$$

Methodology of Graphical Method

Min. $Z = 600X_1 + 500X_2$
Subject to constraints:
$2X_1 + 1X_2 \geq 80$ …….PI
$1X_1 + 1X_2 \geq 60$ …….PII
$X_1, X_2 \geq 0$

Coordinates for constraint lines :

(1) $2X_1 + 1X_2 = 80$ …….PI……. Converting in equality

If $X_2 = 0$, $X_1 = 40$, _ _ _ _ _ (40,0)

If $X_1 = 0$, $X_2 = 80$, _ _ _ _ _ (0,80)

(2) $1X_1 + 2X_2 = 60$ …….PII……. Converting in equality

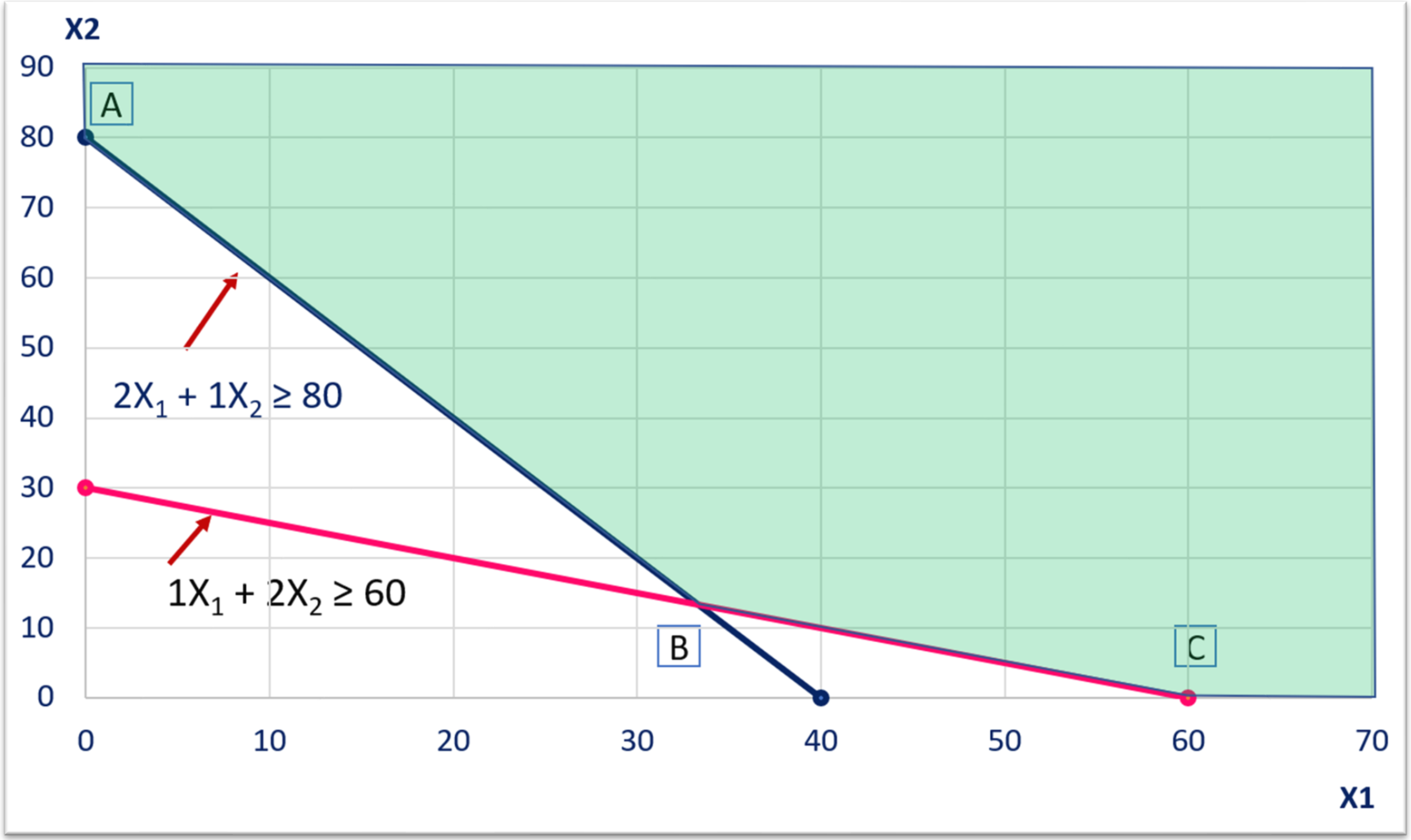
If $X_2 = 0$, $X_1 = 60$, _ _ _ _ _ (60,0)

If $X_1 = 0$, $X_2 = 30$, _ _ _ _ _ (0,30)

Methodology of Graphical Method

To select scale:
Maximum value for X_1 = 60
Maximum value for X_2 = 80
Scale: 1 cm = 10 units

Feasible Region: ABC



Methodology of Graphical Method

Corner point method:		
Vertex	**Co-ordinates**	**Z = $600X_1 + 500X_2$**
A	$X_1 = 0; X_2 = 80$	Z =Rs.40,000
	From Graph	
B	$X_1 = 100/3, X_2 = 40/3$	Z = Rs. 80,000/3
	From simultaneous equations	
C	$X_1 = 60, X_2 = 0$	Z = Rs.36,000
	From Graph	

Min. Z = Rs. 80,000/3 (At point B)

Solution:
Optimal cost = Min Z = Rs. 80,000/3
X_1 = No. of hardcover copies (in a lot of 100) = 100/3
X_2 = No. of paperback copies (in a lot of 100) = 40/3

For B→B is at intersection of two constraint lines
$2X_1 + 1X_2 = 80$ and $1X_1 + 2X_2 = 60$
$2X_1 + 1X_2 = 80$ ……(1)
$1X_1 + 2X_2 = 60$ …..(2)

$2X_1 + 4X_2 = 120$ …..(2) x 2…(3)

Therefore, $3X_2 = 40$ …..(3) - 2

Therefore, $X_2 = 40/3$
Substituting value in equation (1),
$2X_1 + (1*40/3) = 80$
$2X_1 = 80 - 40/3$
$2X_1 = 200/3$
$X_1 = 200/6$
$X_1 = 100/3$