Linear Programming I:
Formulation and Graphic Solution

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Components of an LPP

- Objective Function
  - To maximise or minimise
- Constraints
  - Involving ≤, =, or ≥ sign
  - Usually, a maximisation problem has ≤ type of constraints and a minimisation problem has ≥ type. But a given problem can have constraints involving any of the signs.
- Non-negativity Condition
  - Variables to be non-negative

A Typical LPP

Maximize $Z = 50x_1 + 75x_2 + 60x_3$

Subject to

$$5x_1 + 8x_2 + 7x_3 \leq 480$$
$$4x_1 + 2x_2 + 3x_3 \leq 240$$
$$x_1 - 2x_2 + x_3 \geq 20$$
$$x_1, x_2, x_3 \geq 0$$

Constraints

Objective Function

Non-negativity condition

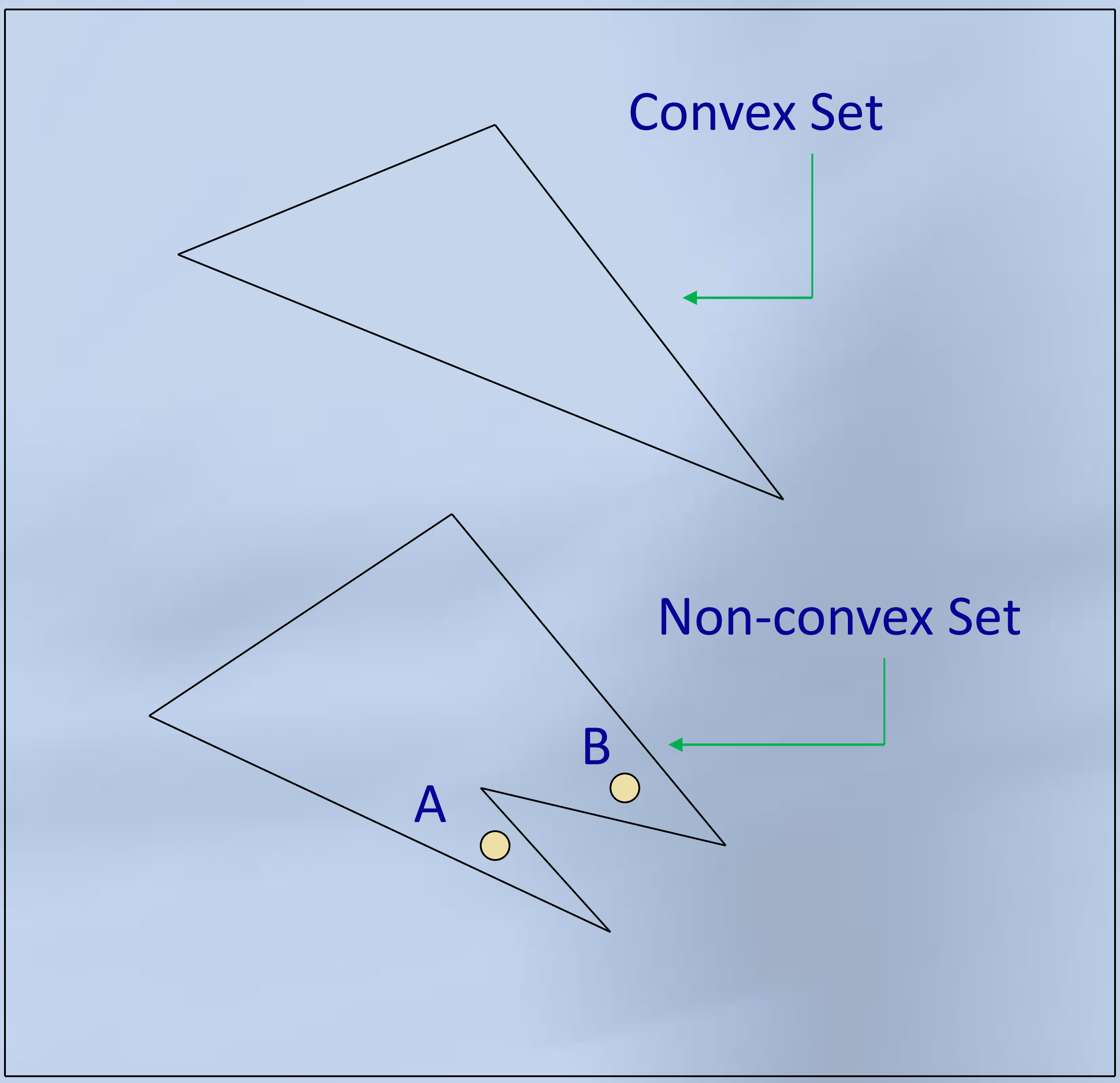
Assumptions underlying Linear Programming

- Proportionality
- Additivity
- Continuity
- Certainty
- Finite Choices

Graphic Solution to LPPs

- Plot constraints – every constraint is represented by a straight line
- Mark feasible region which should be a convex set
- Evaluate corner points/use iso-profit or iso-cost lines to get optimal solution
  - Redundant constraints
  - Binding constraints
  - Non-binding constraints

Convex and Non-convex Sets



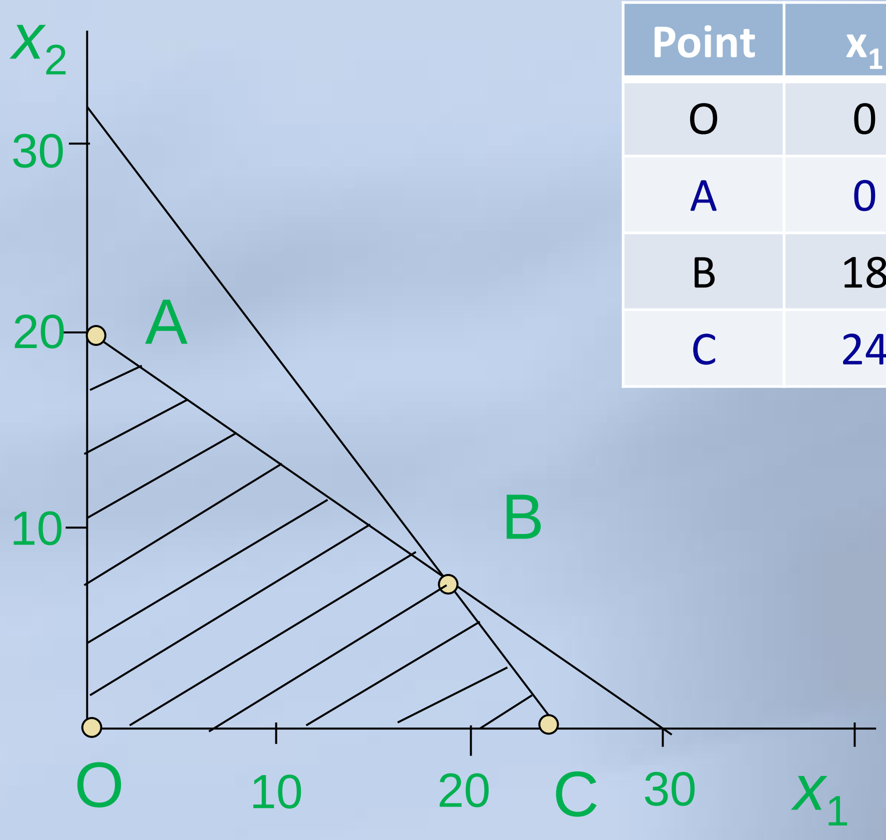
Graphic Solution: Max Problem

Maximize $Z = 40x_1 + 35x_2$

Subject to $2x_1 + 3x_2 \leq 60$

$4x_1 + 3x_2 \leq 96$

$x_1, x_2 \geq 0$

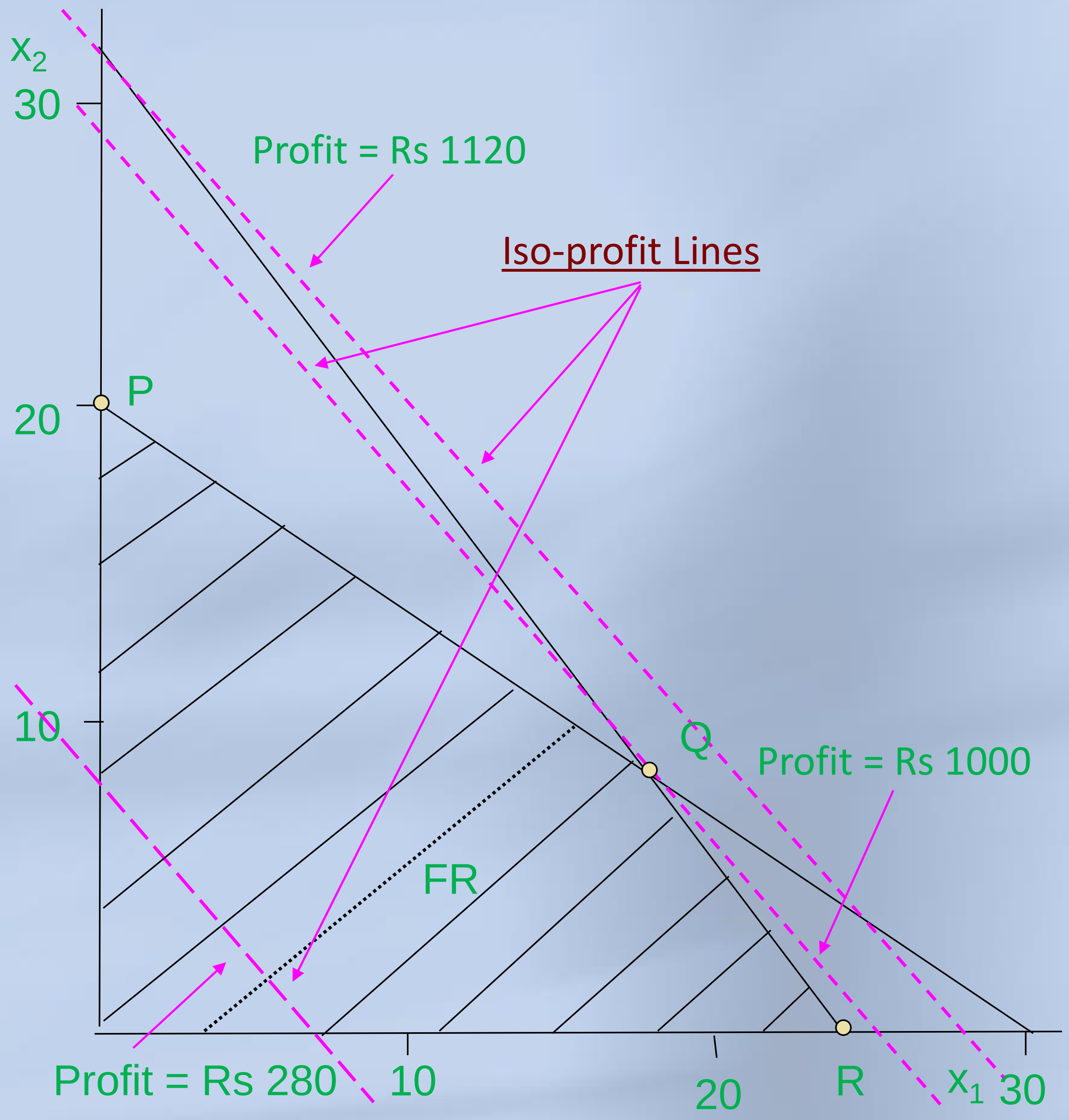


Point	x_1	x_2	Z
O	0	0	0
A	0	20	700
B	18	8	1000
C	24	0	960

Optimal Solution (unique)

Graphic Solution: Max Problem

(using Iso-profit Lines)



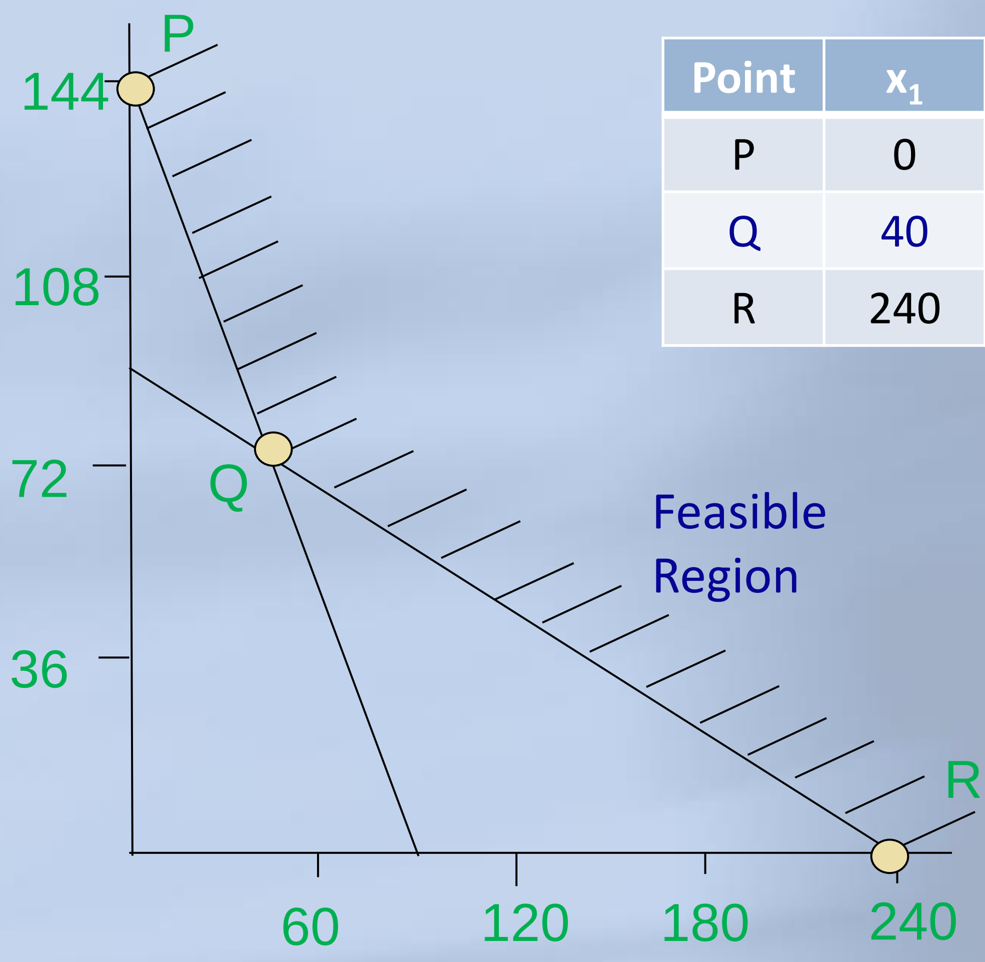
Graphic Solution: Min Problem

Minimize $Z = 40x_1 + 24x_2$

Subject to $20x_1 + 50x_2 \geq 4800$

$80x_1 + 50x_2 \geq 7200$

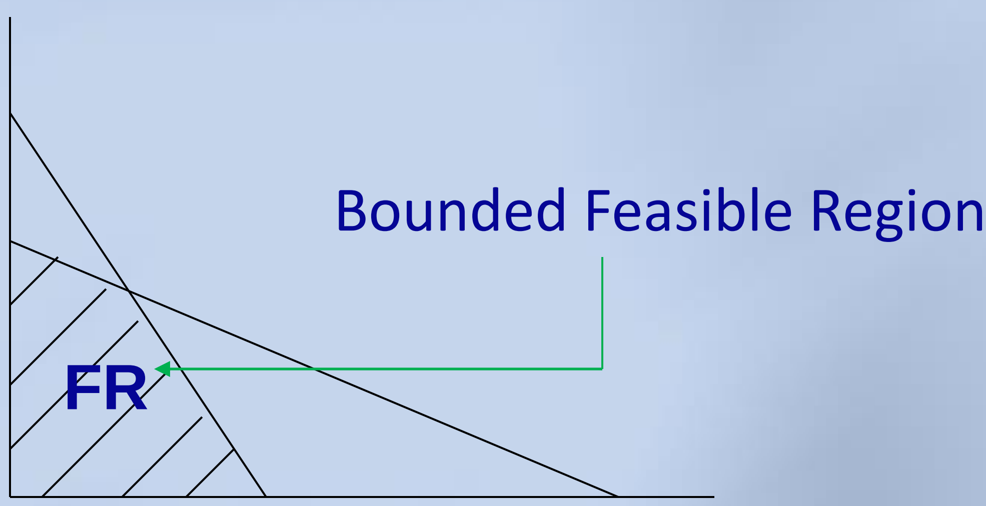
and $x_1 \; x_2 \geq 0$

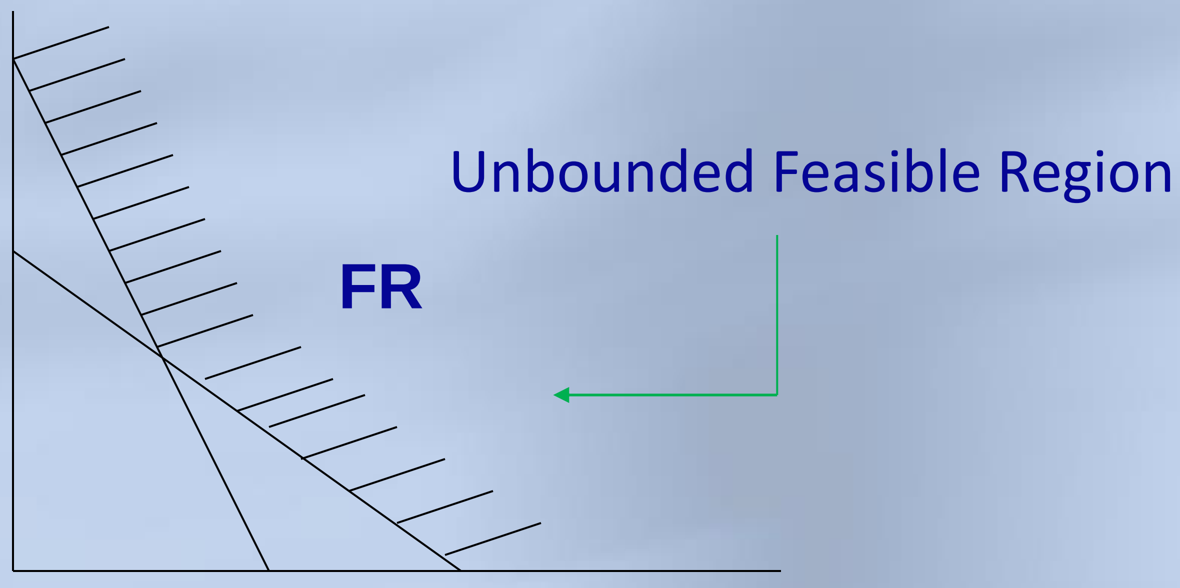


Point	x_1	x_2	Z
P	0	144	3456
Q	40	20	3520
R	240	8	9600

Optimal Solution

Bounded and Unbounded Feasible Regions





Redundant Constraints

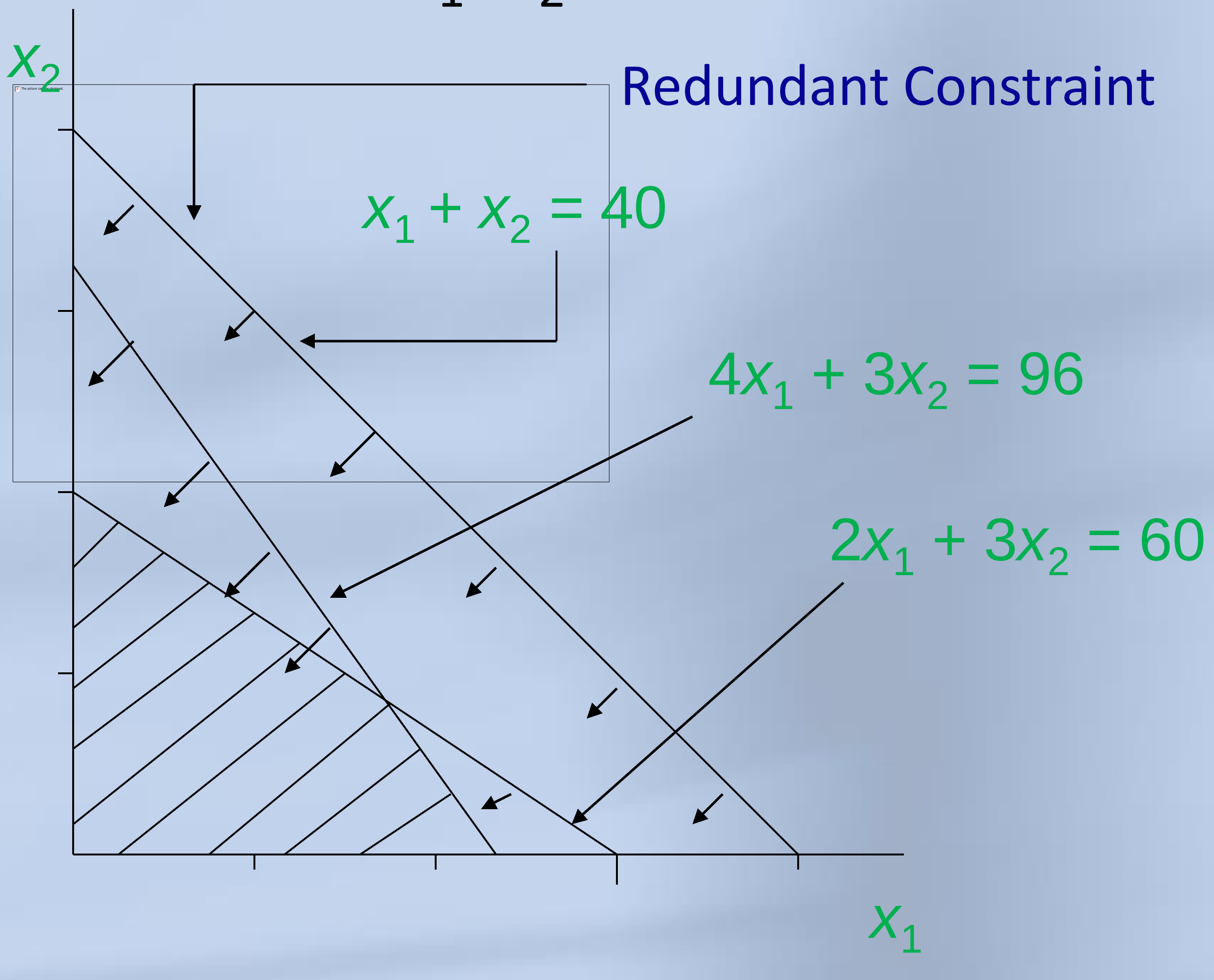
Minimize $Z = 40x_1 + 35x_2$

Subject to $x_1 + x_2 \leq 40$

$4x_1 + 3x_2 \leq 96$

$2x_1 + 3x_2 \leq 60$

$x_1, x_2 \geq 0$



Binding and Non-binding Constraints

- **Binding Constraint**: If the LHS is equal to RHS when optimal values of the decision variables are substituted in to the constraint
- **Non-binding Constraint**: If LHS ≠ RHS on such substitution of optimal values
- For min problem solution,

  $20 \times 0 + 50 \times 144 = 7200 \neq 4800$ (RHS)

  $80 \times 0 + 50 \times 144 = 7200$ (=RHS)

Binding constraint

Non-Binding constraint

Solutions to LPPs

- Unique Optimal Solution
- Multiple Optimal Solutions
- Infeasibility: No feasible solution
- Unbounded Solution

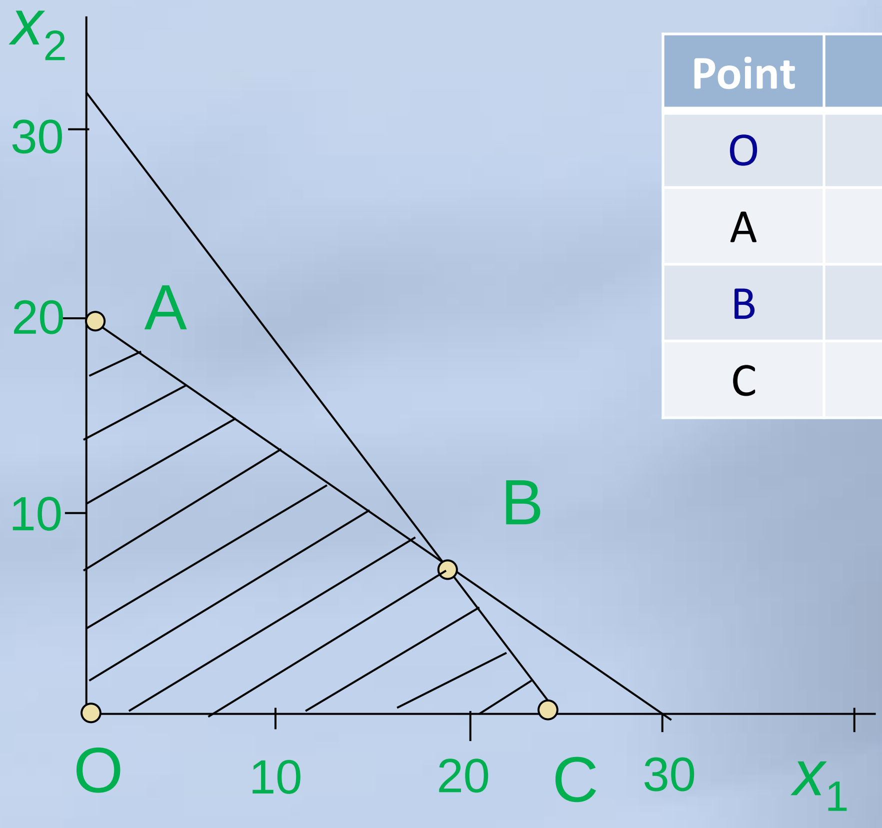
Multiple Optimal Solutions

Maximise $Z = 40x_1 + 30x_2$

Subject to $2x_1 + 3x_2 \leq 60$

$x_1 + 3x_2 \leq 96$

$x_1, x_2 \geq 0$



Point	x_1	x_2	Z
O	0	0	0
A	0	20	600
B	18	8	960
C	24	0	960

Optimal Solution
(Multiple)

Infeasibility: No Feasible Solution

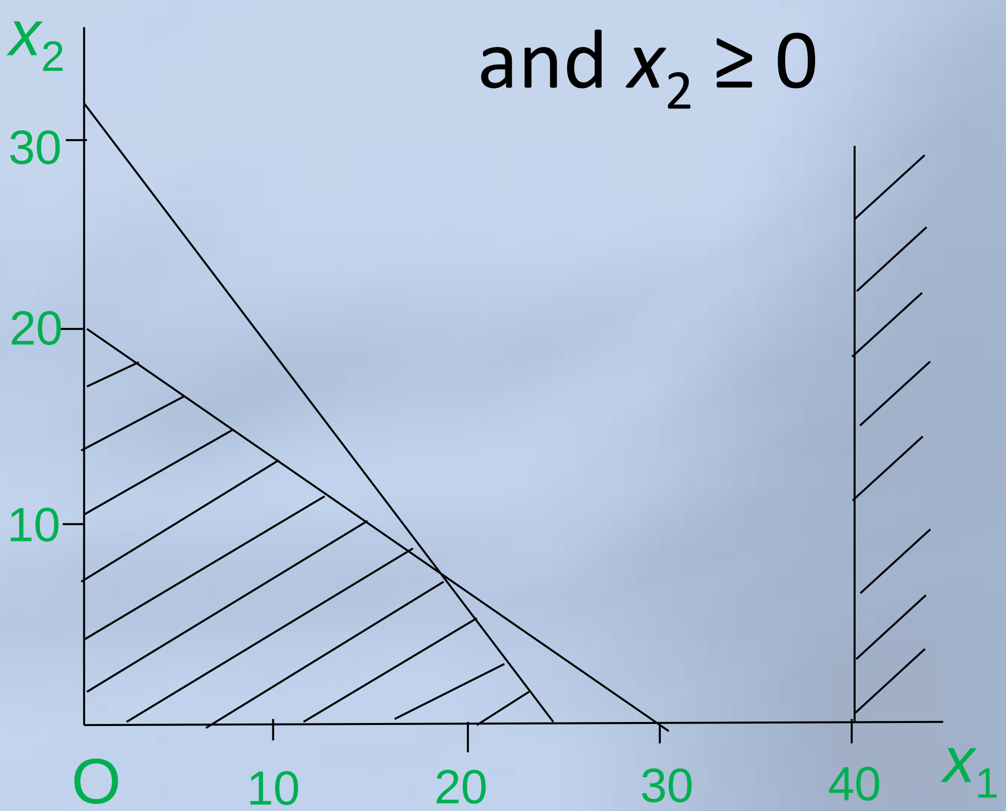
Maximise $Z = 40x_1 + 35x_2$

Subject to $2x_1 + 3x_2 \leq 60$

$x_1 + 3x_2 \leq 96$

$x_1 \geq 40$

and $x_2 \geq 0$



No common point in feasible regions of the constraints

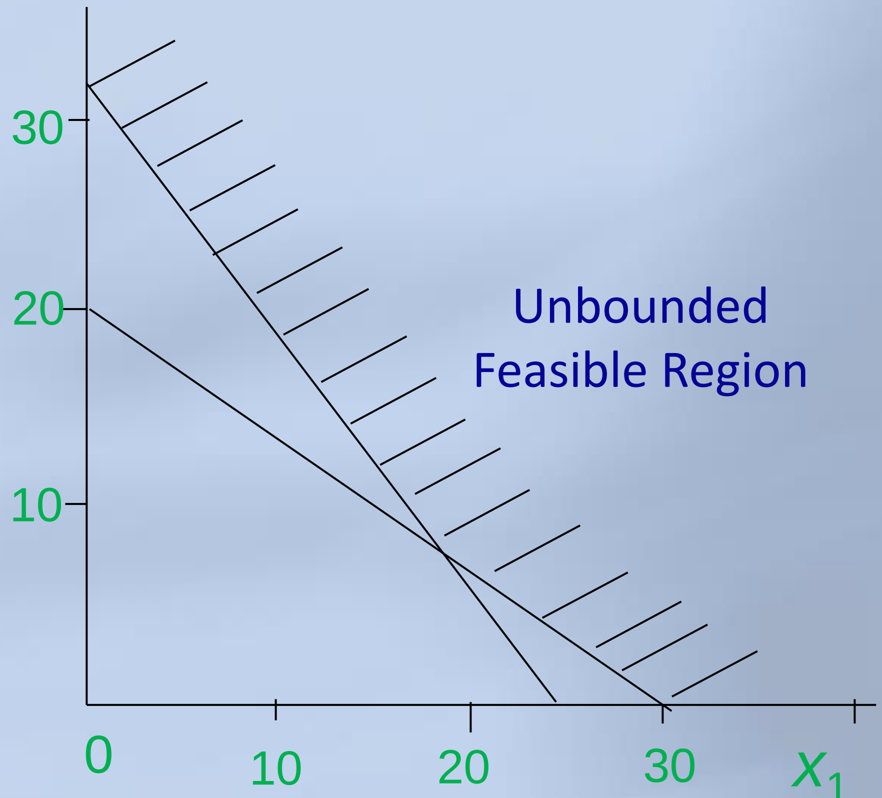
Unbounded Solution

Maximize $Z = 40x_1 + 35x_2$

Subject to $2x_1 + 3x_2 \geq 60$

$4x_1 + 3x_2 \geq 96$

$x_1, x_2 \geq 0$



Multiple Choice Questions

- Which of the following is not true about LPP?
  1. The maximisation or minimisation of some function is the objective.
  2. There are restrictions (i.e. constraints) which limit the degree to which the objective can be pursued.
  3. The constraints must all be $\geq$ or $\leq$ type.
  4. All relationships are linear in nature.

Multiple Choice Questions

- Mark the wrong statement:
  1. An LPP with only two decision variables can be solved using graphic approach.
  2. Every point (x1, x2) on the graph corresponds to a possible solution.
  3. The point (0,0) also represents a solution point.
  4. Only those problems can be solved graphically where the number of constraints is not more than four.

Multiple Choice Questions

- Mark the incorrect statement:
  1. Proportionality
  2. Uncertainty
  3. Additivity
  4. Divisibility

Multiple Choice Questions

- Which of the following is not associated with LPP?
  1. Proportionality A feasible solution satisfies all constraints.
  2. An infeasible solution is one that fails to satisfy all constraints of the problem.
  3. The feasible solution which optimises is called optimal solution.
  4. Keeping in view the assumptions underlying, it is not possible to identify optimal solution to an LPP by trial and error.

Multiple Choice Questions

- The extreme points of feasible region of a maximising LPP, wherein two decision variables x1 and x2 have objective function co-efficients in the ratio 1:3, are O(0, 0), A(708, 0), B(540, 252), C(300, 420) and D(0, 540). What is the optimal solution?

1. x1 = 708, x2 = 0
2. x1 = 540, x2 = 252
3. x1 = 0, x2 = 540
4. x1 = 300, x2 = 420

Multiple Choice Questions

- Which of the following is not true about infeasibility?
  1. It implies that the problem has no feasible solution.
  2. It is independent of the objective function.
  3. It is seen when there is no common point in the feasible regions of all the constraints.
  4. It cannot be detected in graphical solution an LPP.

Multiple Choice Questions

- Mark the wrong statement:
  1. It is possible for a constraint to be of no consequence in determining feasible region of an LPP.
  2. Two constraints of a maximising LPP are given as: (i) $2x1 + 3x2 \leq 18$ and (ii) $2x1 + 4x2 \leq 28$. Constraint (i) is redundant.
  3. An LPP has only two constraints: $x1 + 3x2 \leq 6$ and $2x1 + 4x2 \geq 20$, besides $x1, x2 \geq 0$. It would not have an optimal solution.
  4. A minimisation problem always has unbounded feasible region.

Multiple Choice Questions

- Point out the wrong statement:
  1. The feasible region for an LPP has to be a convex set.
  2. The optimal solution, if present, to an LPP always lies at an extreme point of the feasible region.
  3. The optimal solution obtained by iso-profit/ iso-cost line identical to the optimal solution obtained by evaluating corner points.
  4. A convex set must be bound from all sides.

Multiple Choice Questions

- Mark the wrong statement:
  1. If optimal solution to an LPP exists, it would be unique if the slope of the iso-profit/cost line does not match with the slope of any of the constraints.
  2. An LPP can have multiple optimal solutions.
  3. Different optimal solutions to an LPP can have different objective function values.
  4. A minimisation problem with non-negative variables cannot have unbounded solution.

Multiple Choice Questions

- Identify the wrong statement:
  1. In an LPP to maximise Z = 10x1 + 20x2, a constraint 3x1 + 7x2 ≤ 99 is binding if optimal solution is x1 = 5 and x2 = 12.
  2. In an LPP with unbounded solution, it is possible to change the objective function in such a way that the revised problem has a bounded, optimal solution.
  3. An LPP with unbounded feasible region, would obviously have unbounded solution.
  4. It is possible that all constraints of an LPP to be binding.