OPERATIONS RESEARCH

Introduction to Operations Research

Topic 1

ITM BUSINESS SCHOOL
OPERATIONS AND SUPPLY CHAIN MANAGEMENT
SEMESTER II
VIJAYANTA PAWASE

Course Structure

OPERATIONS RESEARCH: 50 Marks

- ✓ **16 Regular Sessions**

External 30 marks
✓ End term exam
Internal 20 marks

Course Structure

Evaluation Method	Marks
Class Attendance	5
Class Participation	5
Class Test	5
Project /Assignment	5
Total	**20**

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The Origins of Operations Research

- A related problem is that as the complexity and specialization in an organization increase, it becomes more and more difficult to allocate the available resources to the various activities in a way that is most effective for the organization as a whole.

- These kinds of problems and the need to find a better way to solve them provided the environment for the emergence of operations research (commonly referred to as OR).

- The beginning of the activity called operations research has generally been attributed to the military services early in World War II. Because of the war effort, there was an urgent need to allocate scarce resources to the various military operations and to the activities within each operation in an effective manner.

- Therefore, the British and then the U.S. military management called upon a large number of scientists to apply a scientific approach to dealing with this and other strategic and tactical problems.

The Origins of Operations Research

- In effect, they were asked to do research on (military) operations. These teams of scientists were the first OR teams.

- By developing effective methods of using the new tool of radar, these teams were instrumental in winning the Air Battle of Britain.

- Through their research on how to better manage convoy and antisubmarine operations, they also played a major role in winning the Battle of the North Atlantic. Similar efforts assisted the Island Campaign in the Pacific.

- When the war ended, the success of OR in the war effort spurred interest in applying OR outside the military as well. As the industrial boom following the war was running its course, the problems caused by the increasing complexity and specialization in organizations were again coming to the forefront.

The Origins of Operations Research

- It was becoming apparent to a growing number of people, including business consultants who had served on or with the OR teams during the war, that these were basically the same problems that had been faced by the military but in a different context.

- By the early 1950s, these individuals had introduced the use of OR to a variety of organizations in business, industry, and government.

The Origins of Operations Research

One

- The substantial progress that was made early in improving the techniques of OR. After the war, many of the scientists who had participated on OR teams or who had heard about this work were motivated to pursue research relevant to the field; important advancements in the state of the art resulted. A prime example is the simplex method for solving linear programming problems, developed by George Dantzig in 1947. Many of the standard tools of OR, such as linear programming, dynamic programming, queueing theory, and inventory theory, were relatively well developed before the end of the 1950s.

Second

- A second factor that gave great impetus to the growth of the field was the onslaught of the computer revolution. A large amount of computation is usually required to deal most effectively with the complex problems typically considered by OR. Doing this by hand would often be out of the question. Therefore, the development of electronic digital computers, with their ability to perform arithmetic calculations millions of times faster than a human being can, was a tremendous boon to OR. A further boost came in the 1980s with the development of increasingly powerful personal computers accompanied by good software packages for doing OR. This brought the use of OR within the easy reach of much larger numbers of people, and this progress further accelerated in the 1990s and into the 21st century. For example, the widely used spreadsheet package, Microsoft Excel, provides a Solver that will solve a variety of OR problems. Today, literally millions of individuals have ready access to OR software. Consequently, a whole range of computers from mainframes to laptops now are being routinely used to solve OR problems, including some of the enormous size.

The Nature of Operations Research

- As its name implies, operations research involves **"research on operations."**

- Thus, operations research is **applied to problems that concern how to conduct and coordinate the operations (i.e., the activities) within an organization**.

- The **nature of the organization is essentially immaterial**, and in fact, OR has been applied extensively in such diverse areas as manufacturing, transportation, construction, telecommunications, financial planning, health care, the military, and public services, to name just a few.

The Nature of Operations Research

- The research part of the name means that operations research uses an approach that resembles the way research is conducted in established scientific fields.
- To a considerable extent, the scientific method is used to investigate the problem of concern.
- In fact, the term management science sometimes is used as a synonym for operations research.
- In particular, the process begins by carefully observing and formulating the problem, including gathering all relevant data.

The Nature of Operations Research

- The next step is to construct a scientific (typically mathematical) model that attempts to abstract the essence of the real problem.

- It is then hypothesized that this model is a sufficiently precise representation of the essential features of the situation that the conclusions (solutions) obtained from the model are also valid for the real problem.

- Next, suitable experiments are conducted to test this hypothesis, modify it as needed, and eventually verify some form of the hypothesis. (This step is frequently referred to as model validation.)

- Thus, in a certain sense, operations research involves creative scientific research into the fundamental properties of operations. However, there is more to it than this. Specifically, OR is also concerned with the practical management of the organization.

The Nature of Operations Research

- An additional characteristic is that OR frequently attempts to search for the best solution **(referred to as an optimal solution)** for the model that represents the problem under consideration. (We say a best instead of the best solution because multiple solutions may be tied as best.)

- It must be interpreted carefully in terms of the practical needs of management, this **“search for optimality”** is an important theme in OR.

- Such an OR team typically needs to include individuals who collectively are highly trained in mathematics, statistics and probability theory, economics, business administration, computer science, engineering and the physical sciences, the behavioral sciences, and the special techniques of OR. The team also needs to have the necessary experience and variety of skills to give appropriate consideration to the many ramifications of the problem throughout the organization.

The Impact of Operations Research

Operations research has had an impressive impact on improving the efficiency of numerous organizations around the world.

Applications of operations research to be described in application vignettes

Organization	Area of Application	Section	Annual Savings
Federal Express	Logistical planning of shipments	1.4	Not estimated
Continental Airlines	Reassign crews to flights when schedule disruptions occur	2.2	$40 million
Swift & Company	Improve sales and manufacturing performance	3.1	$12 million
Memorial Sloan-Kettering Cancer Center	Design of radiation therapy	3.4	$459 million
Welch's	Optimize use and movement of raw materials	3.5	$150,000
INDEVAL	Settle all securities transactions in Mexico	3.6	$150 million
Samsung Electronics	Reduce manufacturing times and inventory levels	4.3	$200 million more revenue
Pacific Lumber Company	Long-term forest ecosystem management	7.2	$398 million NPV
Procter & Gamble	Redesign the production and distribution system	9.1	$200 million
Canadian Pacific Railway	Plan routing of rail freight	10.3	$100 million
Hewlett-Packard	Product portfolio management	10.5	$180 million
Norwegian companies	Maximize flow of natural gas through offshore pipeline network	10.5	$140 million
United Airlines	Reassign airplanes to flights when disruptions occur	10.6	Not estimated
U.S. Military	Logistical planning of Operations Desert Storm	11.3	Not estimated
MISO	Administer the transmission of electricity in 13 states	12.2	$700 million

The Impact of Operations Research

Applications of operations research to be described in application vignettes

Organization	Area of Application	Section	Annual Savings
Netherlands Railways	Optimize operation of a railway network	12.2	$105 million
Taco Bell	Plan employee work schedules at restaurants	12.5	$13 million
Waste Management	Develop a route-management system for trash collection and disposal	12.7	$100 million
Bank Hapoalim Group	Develop a decision-support system for investment advisors	13.1	$31 million more revenue
DHL	Optimize the use of marketing resources	13.10	$22 million
Sears	Vehicle routing and scheduling for home services and deliveries	14.2	$42 million
Intel Corporation	Design and schedule the product line	14.4	Not estimated
Conoco-Phillips	Evaluate petroleum exploration projects	16.2	Not estimated
Workers' Compensation Board	Manage high-risk disability claims and rehabilitation	16.3	$4 million
Westinghouse	Evaluate research-and-development projects	16.4	Not estimated
KeyCorp	Improve efficiency of bank teller service	17.6	$20 million
General Motors	Improve efficiency of production lines	17.9	$90 million
Deere & Company	Management of inventories throughout a supply chain	18.5	$1 billion less inventory

The Impact of Operations Research

Applications of operations research to be described in application vignettes (contd)

Organization	**Area of Application**	**Section**	**Annual Savings**
Time Inc.	Management of distribution channels for magazines	18.7	$3.5 million more profit
InterContinental Hotels	Revenue management	18.8	$400 million more revenue
Bank One Corporation	Management of credit lines and interest rates for credit cards	19.2	$75 million more profit
Merrill Lynch	Pricing analysis for providing financial services	20.2	$50 million more revenue
Sasol	Improve the efficiency of its production processes	20.5	$23 million
FAA	Manage air traffic flows in severe weather	20.5	$200 million

Problem Solving and Decision Making

One way of summarizing the usual (overlapping) phases of an OR study is the following:

1	• Define the problem of interest and gather relevant data.
2	• Formulate a mathematical model to represent the problem.
3	• Develop a computer-based procedure for deriving solutions to the problem from the model.
4	• Test the model and refine it as needed.
5	• Prepare for the ongoing application of the model as prescribed by management.
6	• Implement.

Problem Solving and Decision Making

Problem solving can be defined as the process of identifying a difference between the actual and the desired state of affairs and then taking action to resolve the difference. For problems important enough to justify the time and effort of careful analysis, the problem-solving process involves the following seven steps:

- **7 Steps of Problem Solving**

  (First 5 steps are the process of decision making)

  1. Identify and define the problem.
  2. Determine the set of alternative solutions.
  3. Determine the criteria for evaluating alternatives.
  4. Evaluate the alternatives.
  5. Choose an alternative (make a decision).

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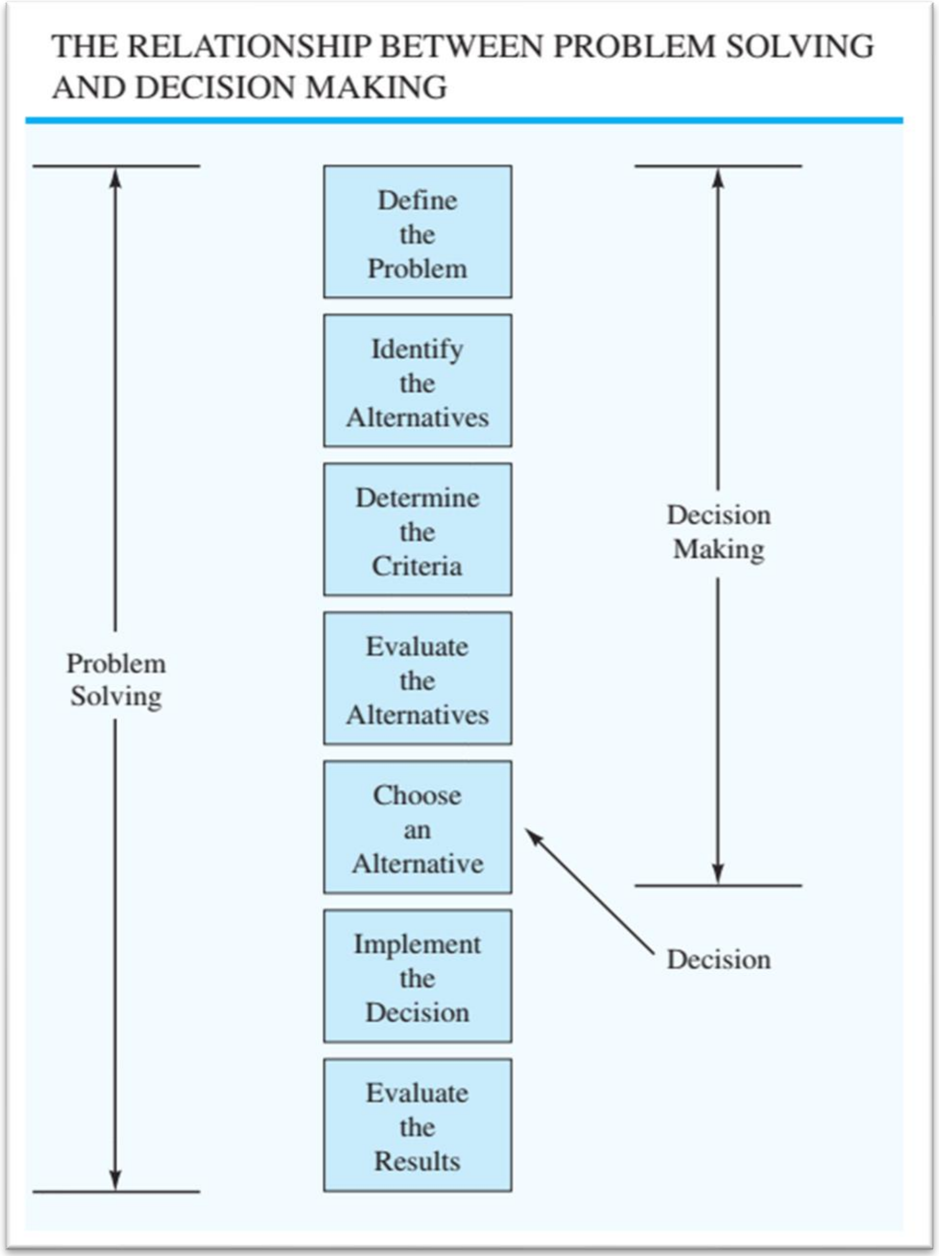
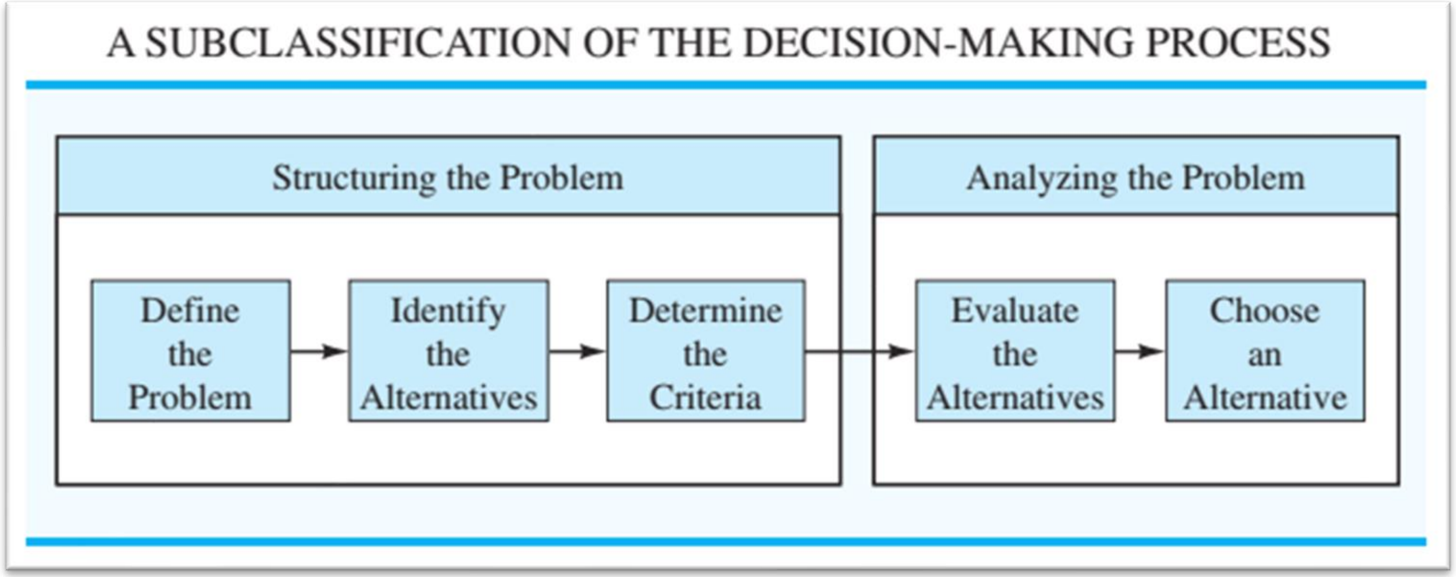
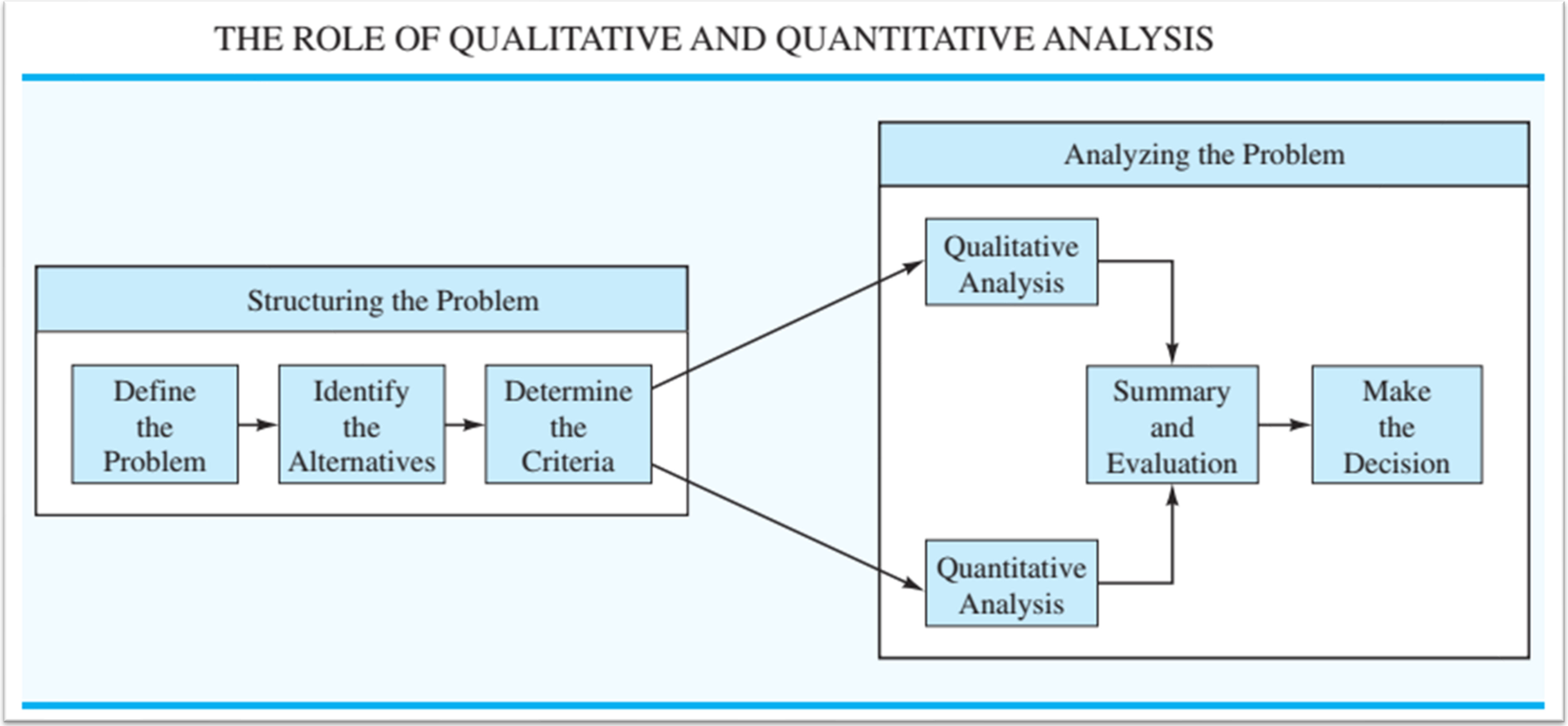
  6. Implement the selected alternative.
  7. Evaluate the results.

Problem Solving and Decision Making

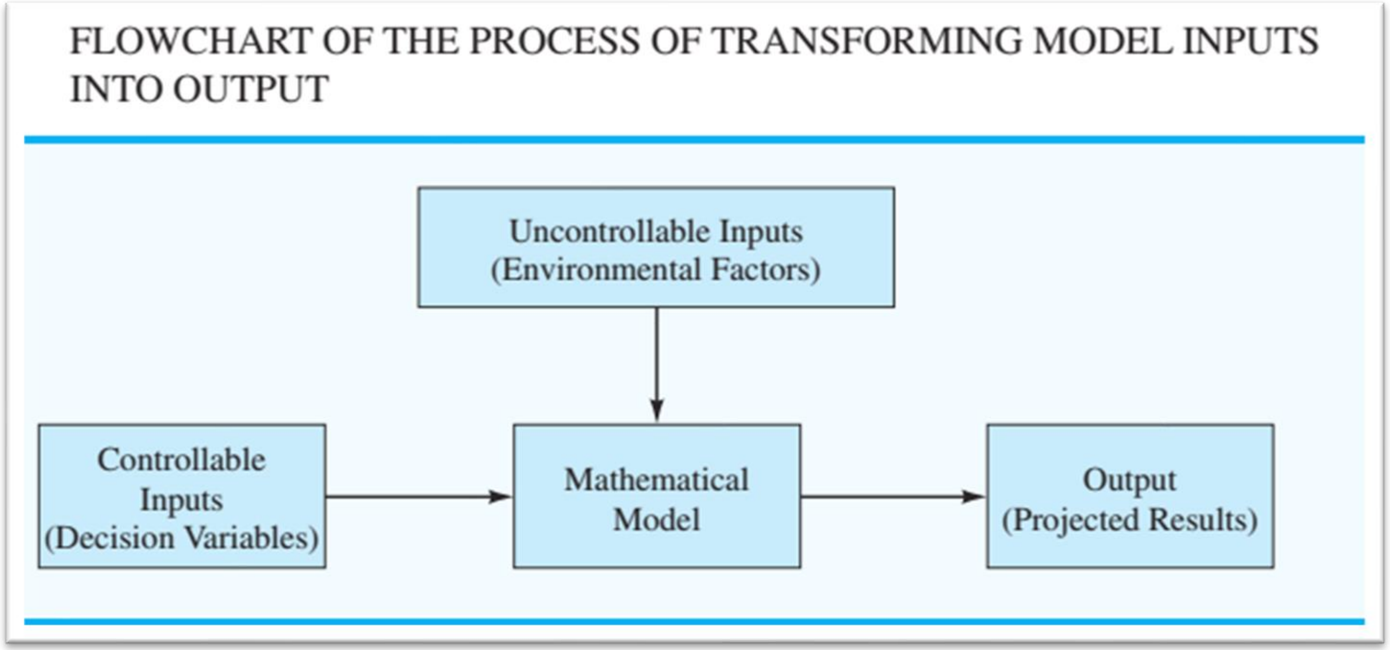




Problem Solving and Decision Making

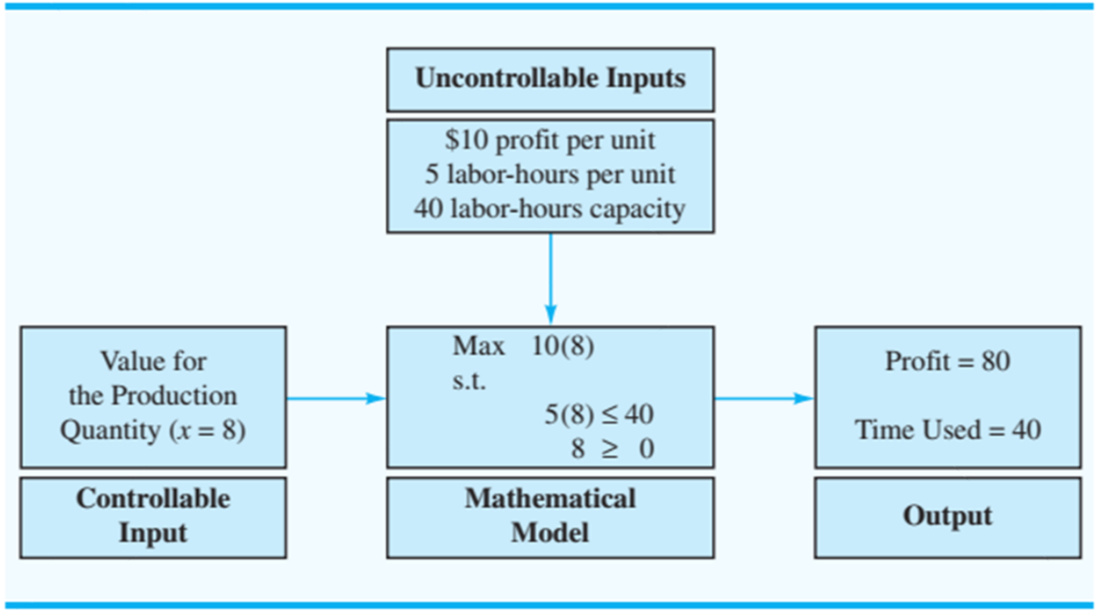


Problem Solving and Decision Making



Problem Solving and Decision Making

FLOWCHART FOR THE PRODUCTION MODEL



TRIAL-AND-ERROR SOLUTION FOR THE PRODUCTION MODEL OF FIGURE FLOWCHART FOR THE PRODUCTION MODEL

Decision Alternative (Production Quantity) x	Projected Profit	Total Hours of Production	Feasible Solution? (Hours Used ≤ 40)
0	0	0	Yes
2	20	10	Yes
4	40	20	Yes
6	60	30	Yes
8	80	40	Yes
10	100	50	No
12	120	60	No

Linear Programming - 1

Basic Concepts and Problem Formulation

Introduction

Linear programming is a technique for determining how to use the limited resources of a business to achieve a specified objective such as

- maximum revenue,
- maximum profit,
- minimum cost,
- minimum time etc.

Hence, linear programming is an optimization technique.

Introduction

A common definition of linear programming is

"It is the analysis of problems in which a linear function of a number of variables is to be maximized or minimized when those variables are subject to a number of constraints(restrictions) or in the form of linear equations and/or inequalities".

Application Areas for Linear Programming

A) Industrial Applications:

Product-mix decisions:

- A company can produce a variety of products having different costs and different selling prices. The decision is to be made to find the optimal profit mix.

Production scheduling decisions:

- If a number of jobs are to be performed on a number of machines, the production schedule needs to be optimized to maximize profit or minimize cost or time. Another form of production scheduling problem is to meet seasonal or fluctuating demand. This may involve the use of overtime or extra capacity during peak demand.

Production distribution problems:

- Different products could be manufactured at different plants at different costs and could be sold at different prices. Hence optimal allocation of production and distribution centres are to be determined.

Application Areas for Linear Programming

B) Management Applications:

Portfolio selection:

- Investment related decisions regarding the selection of specific investments from the wide availability of choices needs to be finalized. The risk and returns associated with each alternative could be different.

Media selection:

- To decide the optimal media mix for the execution of an advertising strategy. This helps in making optimal use of the advertising budget.

Traveling salesman problem:

- To find the shortest route for a salesman who wants to cover a number of cities without visiting any city twice.

Staffing problem:

- To find work schedule for places like restaurants, police stations, etc. where the need for staff may vary throughout the day (peak hours and non-peak hours). To minimize the total number of employees.

Application Areas for Linear Programming

C) Miscellaneous applications:

Airline routing:

- To make the best use of aircrafts and crews, the schedule of airlines should be planned in such a way that the crews and equipment get the desired minimum rest or lay-over time and at the same time flight schedule for 'to & fro' flights are optimized.

Diet problem:

- The objective is to satisfy the minimum nutrition requirements of humans or animals at a minimum cost of the diet.

Farm planning:

- In farming, there will be restrictions on the availability of resources such as land, labor hours, working capital, etc. At the same time, it may be possible to grow a number of crops on the available land where revenue potential and cost for each crop is different. Hence, we have to make decisions regarding the optimal mix of crops.

Terminology of Linear Programming

Decision variables:

- Decision variables are entities (e.g., products, services, etc.) whose values are to be determined from the solution. An optimal solution is the one for which the values of individual variables are such that total profit is the maximum or total cost is minimum.
- The decision variables consume resources. It means they compete against each other for sharing the available limited resources.
- Notation used for decision variables is x_1, x_2, x_3, ..., etc.
- The decision variables have a 'linear' relationship among themselves and, with available resources.

Hence, if a company manufactures three products - televisions, refrigerators, and washing machines, then:

x_1 = no. of units of television sets.

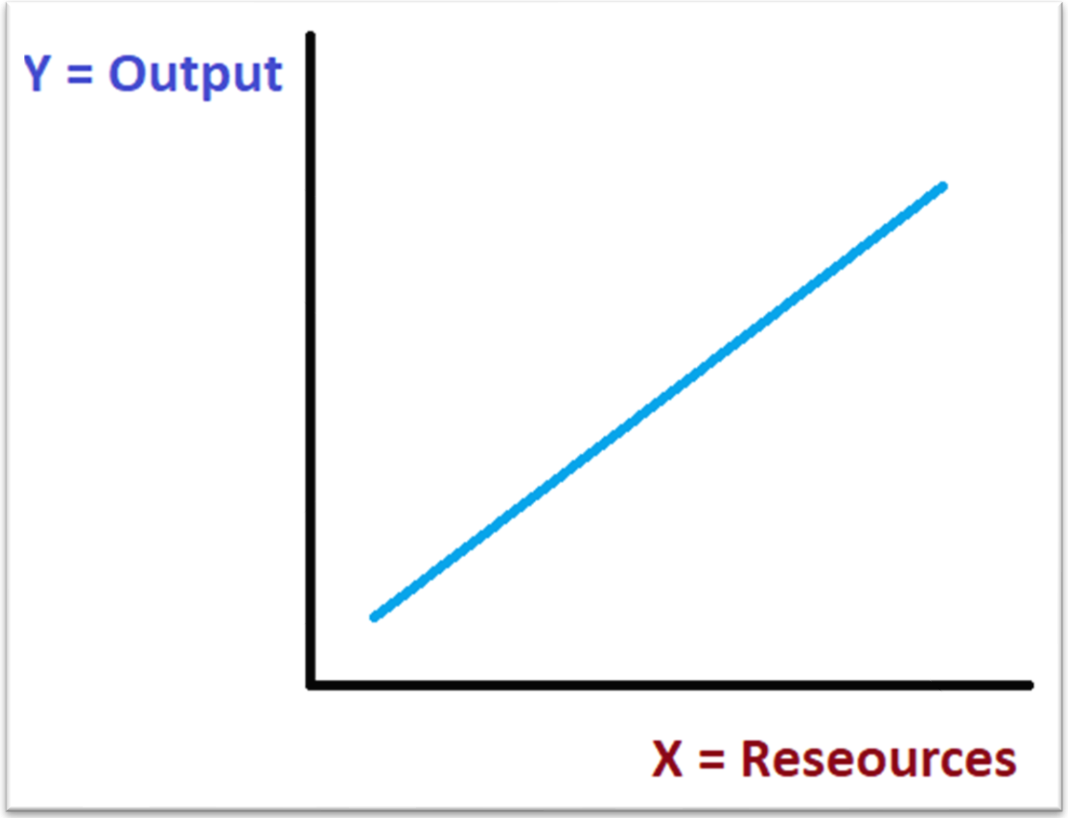
x_2 = no. of units of refrigerators.

x_3 = no. of units of washing machines.

Terminology of Linear Programming

The concept of linearity in linear programming

- Linearity means if resources are increased by 100%, then production will increase by 100%, and hence profit will increase by 100%.
- In LPP, it is assumed that the production or output is directly proportional to the availability of resources or input. Mathematically, we can say Y=f(X) i.e. Y (output) is a linear function of X (input). Y is the dependent variable and X is the independent variable



In other words, the output will increase or decrease in the same proportion of increase or decrease of resources, i.e. if resources are increased by 50%, the output will increase by 50%.

Terminology of Linear Programming

Objective function:

- It specifies the objective of finding the solution to the stated problem and it is expressed in terms of decision variables.
- The objective of the solution could be the maximization of profit or revenue or sales (Max. Z) or minimization of cost or time(Min. Z).
- Coefficients of decision variables in the objective function represent the profit per unit or cost per unit for each decision variable.

In the above example:
If Profit/unit:

- T.V. = Rs. 2,000
- Refrigerator = Rs. 3,000
- Washing machine = Rs. 1,500

Then, objective function will be:

$$\textbf{Max. } Z = 2000x_1 + 3000x_2 + 1500x_3$$

Terminology of Linear Programming

Constraints:

- Constraints are the restrictions or limitations imposed on the problem.
- Constraints can be related to the availability of resources, supply or demand conditions, etc.
- Constraints are of three types:
  (i) Less than or equal to (≤)
  (ii) Greater than or equal to (≥)
  (iii) Equal to (=)

'Less than or equal to' (≤) constraint is associated with the availability of resources or restriction on the production of a Product.

'Greater than or equal to' (≥) constraint is generally associated with achieving minimum supply conditions or minimum consumption conditions.

Terminology of Linear Programming

Example 1:

- In the above example, let us assume that there are three machines M1, M2, and M3 which are used for producing T.V., refrigerator, and washing machines.
- One unit of T.V. requires 2 hours on M1, 3 hours on M2, and 1.5 hours on M3.
- One unit of refrigerator requires 1.5 hours on M1, 2 hours on M2, and 4 hours on M3.
- One unit of washing machine requires 0.5 hours on M1, 1.5 hours on M2, and 0.75 hours on M3.
- Monthly availability of hours on M1, M2, and M3 are 1000 hrs, 1500 hrs, and 3000 hrs respectively.

Terminology of Linear Programming

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Conversion of data in Table Form

Terminology of Linear Programming

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- Monthly availability of hours on M1, M2, and M3 are 1000 hrs, 1500 hrs, and 3000 hrs respectively.

Conversion of data in Table Form

Product	Resources/Unit			
	M1	M2	M3	
T.V.	2	3	1.5	
Refrigerator	1.5	2	4	
Washing M/c	0.5	1.5	0.75	
	1000 Hrs.	1500 Hrs.	3000 Hrs.	← Resources Availability

- There will be 3 capacity constraints, one each for M1, M2 & M3.
- Maximum resource availability is given for M1, M2 & M3.

Terminology of Linear Programming

Example 1:

Conversion of data in Table Form

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	M1	M2	M3	
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Constraints:

(1) For MI

Terminology of Linear Programming

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	1000 Hrs.	1500 Hrs.	3000 Hrs.	← Resources Availability

Constraints:

(1) For MI

$$2x_1 + 1.5x_2 + 0.5x_3 \leq 1000$$

Explanation:

- for 1 unit of T.V., 2 hrs are needed. Hence, for x_1 units of TV. $2x_1$ hrs.
- for 1 unit of a refrigerator, 1.5 hrs are needed. Hence, for x_2 units of refrigerator $1.5x_2$ hrs.
- for 1 unit of a washing machine, 0.5 hrs are needed. Hence, for x_3 units of washing machine $0.5x_3$ hrs.
- Maximum availability = 1000 hrs.

Terminology of Linear Programming

Example 1:

Conversion of data in Table Form

Product	**Resources/Unit**			
	M1	**M2**	**M3**	
T.V.	2	3	1.5	
Refrigerator	1.5	2	4	
Washing M/c	0.5	1.5	0.75	
	1000 Hrs.	1500 Hrs.	3000 Hrs.	← **Resources Availability**

Constraints:

(2) For M2

(3) For M3

(4) Non-negativity constraint:

Terminology of Linear Programming

Example 1:

Conversion of data in Table Form

Product	Resources/Unit			
	M1	M2	M3	
T.V.	2	3	1.5	
Refrigerator	1.5	2	4	
Washing M/c	0.5	1.5	0.75	
	1000 Hrs.	1500 Hrs.	3000 Hrs.	← Resources Availability

Constraints:

(2) For M2

$$3x_1 + 2x_2 + 1.5x_3 \leq 1500$$

(3) For M3

$$1.5\, x_1 + 4x_2 + 0.75x_3 \leq 3000$$

(4) Non-negativity constraint:

$$x_1, x_2, x_3 \geq 0$$

- The production of T.V., refrigerators, and washing machines cannot be negative. It can be positive or zero.

Formulation of Linear Programming Problem(LPP)

The complete formulation:

x_1 = No. of units of television sets/month.

x_2 = No. of refrigerators/month.

x_3 = No. of washing machines/month.

Max. $Z = 2000x_1 + 3000x_2 + 1500x_3$

Subject to constraints:

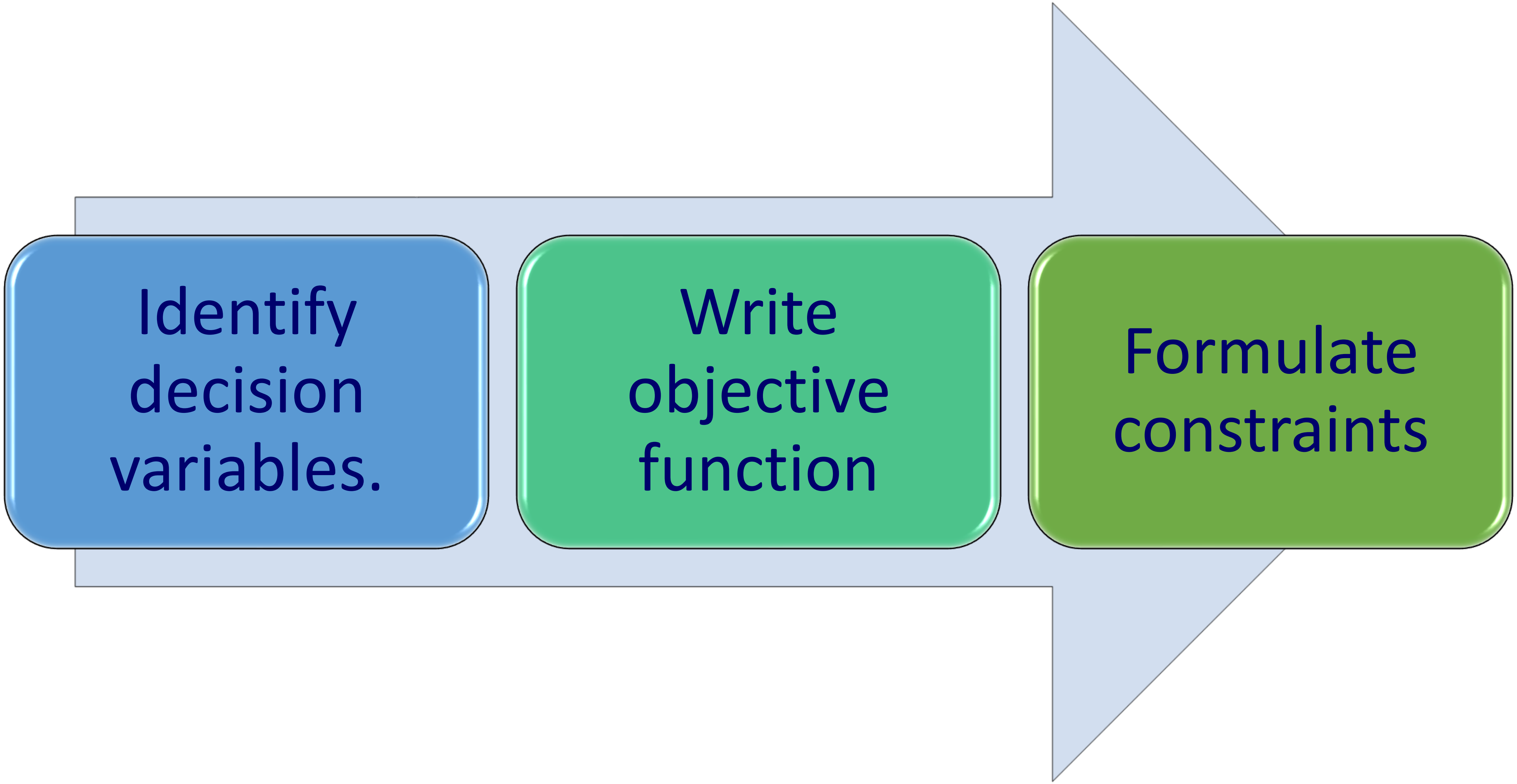
$2x_1 + 1.5x_2 + 0.5x_3 \leq 1000$

$3x_1 + 2x_2 + 1.5x_3 \leq 1500$

$1.5x_1 + 4x_2 + 0.75x_3 \leq 3000$

$x_1, x_2, x_3 \geq 0$

Formulation of Linear Programming Problem(LPP)



Formulation of Linear Programming Problem(LPP)

Example 2 - Diet Problem:

- Vitamins B_1 and B_2 are found in two foods F_1 and F_2.
- 1 unit of F_1, contains 3 units of B_1 and 4 units of B_2.
- 1 unit of F_2 contains 5 units of B_1 and 3 units of B_2.
- The minimum daily prescribed consumption of B_1 & B_2 is 50 and 60 units respectively.
- Cost per unit of F_1 & F_2 is Rs. 6 & Rs. 3 respectively.
- Formulate as LPP.

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Solution:

Tabular Form

	Foods		

Decision variables:

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Solution:

Tabular Form

Vitamins	**Foods**		**Minimum Consumption**
	F_1	**F_2**	
B_1	3	5	50
B_2	4	3	60

Decision variables:

x_1 = No. of units of F1 per day.

x_2 = No. of units of F2 per day.

Formulation of Linear Programming Problem(LPP)

Example 2 - Diet Problem:

- Vitamins B_1 and B_2 are found in two foods F_1 and F_2.
- 1 unit of F_1, contains 3 units of B_1 and 4 units of B_2.
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- Formulate as LPP.

Tabular Form

Vitamins	Foods		Minimum Consumption
	F_1	F_2	
B_1	3	5	50
B_2	4	3	60

Solution:

Decision variables:

x_1 = No. of units of F1 per day.

x_2 = No. of units of F2 per day.

Objective function: (Minimize Cost)

Formulation of Linear Programming Problem(LPP)

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Vitamins	**Foods**		**Minimum Consumption**
	F_1	**F_2**	
B_1	3	5	50
B_2	4	3	60

Solution:

Decision variables:

x_1 = No. of units of F1 per day.

x_2 = No. of units of F2 per day.

Objective function: (Minimize Cost)

$$\text{Min. } Z = 6x_1 + 3x_2$$

Formulation of Linear Programming Problem(LPP)

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Solution:

Decision variables:

x_1 = No. of units of F1 per day.

x_2 = No. of units of F2 per day.

Objective function: (Minimize Cost)

$$\text{Min. } Z = 6x_1 + 3x_2$$

Subject to constraints:

Formulation of Linear Programming Problem(LPP)

Example 2 - Diet Problem:

- Vitamins B_1 and B_2 are found in two foods F_1 and F_2.
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- Formulate as LPP.

Tabular Form

Vitamins	Foods		Minimum Consumption
	F_1	F_2	
B_1	3	5	50
B_2	4	3	60

Solution:

Decision variables:

x_1 = No. of units of F1 per day.

x_2 = No. of units of F2 per day.

Objective function: (Minimize Cost)

$$\text{Min. } Z = 6x_1 + 3x_2$$

Subject to constraints:

$$3x_1 + 5x_2 \geq 50 \quad \text{......(Min. supply of } B_1)$$

$$4x_1 + 3x_2 \geq 60 \quad \text{......(Min. supply of } B_2)$$

$$x_1, x_2 \geq 0$$

Note: Fl & F2 are decision variables and not B1 & B2 because the cost is associated with F1 & F2.

Formulation of Linear Programming Problem(LPP)

Example 3 - Portfolio Selection (Investment decisions)

- An investor is considering investing in two securities 'A' and 'B'. The risk and return associated with these securities are different.
- Security' A' gives a return of 9% and has a risk factor of 5 on a scale of zero to 10. Security 'B' gives a return of 15% but has a risk factor of 8.
- The total amount to be invested is Rs. 5,00,000. Total minimum returns on the investment should be 12%. Maximum combined risk should not be more than 6.
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Solution:

Decision Variables:

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The objective is to maximize the return on total investment.

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- Formulate as LPP.

Solution:

Decision Variables:

x_1 = Amount invested in Security A

x_2 = Amount invested in Security B

Objective Function:

The objective is to maximize the return on total investment.

Therefore, **Max** $\mathbf{Z = 0.09x_1 + 0.15x_2}$ **(9% = 0.09, 15%= 0.15)**

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Objective Function:

The objective is to maximize the return on total investment.

Therefore, **Max $Z = 0.09\, x_1 + 0.15 x_2$ (9% = 0.09, 15%= 0.15)**

Constraints:

(1) Related to Total Investment:

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Constraints:

(1) Related to Total Investment:

$$x_1 + x_2 = 5,00,000$$

Formulation of Linear Programming Problem(LPP)

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Constraints:

(1) Related to Total Investment:

$$x_1 + x_2 = 5{,}00{,}000$$

(2) Related to Risk:

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Therefore, **Max Z = 0.09 x_1 + 0.15x_2 (9% = 0.09, 15%= 0.15)**

Constraints:

(1) Related to Total Investment:

$$x_1 + x_2 = 5,00,000$$

(2) Related to Risk:

$$5x_1 + 8x_2 \leq [6 \times 5,00,000]$$

Therefore, $$5x_1 + 8x_2 \leq 30,00,000$$

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Constraints:

(1) Related to Total Investment:

$x_1 + x_2 = 5,00,000$

(2) Related to Risk:

$5x_1 + 8x_2 \leq [6 \times 5,00,000]$

Therefore, $5x_1 + 8x_2 \leq 30,00,000$

(3) Related to Returns:

(4) Non - negativity:

$x_1, x_2 \geq 0$

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The objective is to maximize the return on total investment.

Therefore, **Max Z = $0.09\,x_1 + 0.15x_2$ (9% = 0.09, 15%= 0.15)**

Constraints:

(1) Related to Total Investment:

$$x_1 + x_2 = 5{,}00{,}000$$

(2) Related to Risk:

$$5x_1 + 8x_2 \leq [6 \text{ x } 5{,}00{,}000]$$

Therefore, $5x_1 + 8x_2 \leq 30{,}00{,}000$

(3) Related to Returns:

$$0.09x_1 + 0.15x_2 \geq [0.12 \text{ x } 5{,}00{,}000]$$

$$0.09x_1 + 0.15x_2 \geq 60{,}000$$

(4) Non - negativity:

$$x_1, x_2 \geq 0$$

Formulation of Linear Programming Problem(LPP)

Example 4 - Media Selection:

- An advertising agency is planning to launch an ad campaign. Media under consideration are T.V., Radio & Newspaper.
- Each medium has a different reach potential and different costs.
- Minimum 10,000,000 households are to be reached through T.V.
- Expenditure on newspapers should not be more than Rs. 10.00 lacs. The total advertising budget is Rs. 20 million.
- The following data is available:

Medium	Cost per unit (Rs.)	Reach per unit (No. of households)
T.V.	200,000	2000,000
Radio	80,000	1000,000
Newspaper	40,000	200,000

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Solution:

Decision Variables:

x_1 = No. of units of T.V. ads.

x_2 = No. of units of Radio ads.

x_3 = No. of units of Newspaper ads.

Formulation of Linear Programming Problem(LPP)

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Decision Variables:

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x_2 = No. of units of Radio ads.

x_3 = No. of units of Newspaper ads.

Objective function: (Maximize reach)

Subject to constraints:

Formulation of Linear Programming Problem(LPP)

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Solution:

Decision Variables:
x_1 = No. of units of T.V. ads.
x_2 = No. of units of Radio ads.
x_3 = No. of units of Newspaper ads.

Objective function: (Maximize reach)
Max. $Z = 2000{,}000x_1 + 1000{,}000x_2 + 200{,}000x_3$

Subject to constraints:

$2000{,}000x_1 \geq 10{,}000{,}000$ ...... (for T.V.)
$40{,}000x_3 \leq 1000{,}000$ .... (for Newspaper)
$200{,}000x_1 + 80{,}000x_2 + 40{,}000x_3 \leq 20{,}000{,}000$ ...... (Ad. budget)
$x_1, x_2, x_3 \geq 0$

Formulation of Linear Programming Problem(LPP)

Example 5 - Product Mix Problem:

- A factory manufactures two products A and B.
- To manufacture one unit of A, 1.5 machine hours and 2.5 labor hours are required.
- To manufacture product B, 2.5 machine hours and 1.5 labor hours are required.
- In a month, 300 machine hours and 240 labor hours are available.
- Profit per unit, for A is Rs. 50 and for B is Rs. 40.
- Formulate as LPP.

Formulation of Linear Programming Problem(LPP)

Example 5 - Product Mix Problem:

- A factory manufactures two products A and B.
- To manufacture one unit of A, 1.5 machine hours and 2.5 labor hours are required.
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- Formulate as LPP.

Solution:

Tabular Form		

Formulation of Linear Programming Problem(LPP)

Example 5 - Product Mix Problem:

- A factory manufactures two products A and B.
- To manufacture one unit of A, 1.5 machine hours and 2.5 labor hours are required.
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- Profit per unit, for A is Rs. 50 and for B is Rs. 40.
- Formulate as LPP.

Solution:

Tabular Form

Products	Resources/unit	
	Machine	Labor
A	1.5	2.5
B	2.5	1.5
Availability	300 Hrs	240 Hrs

There will be two constraints. One for machine hours availability and for labor hours availability.

Formulation of Linear Programming Problem(LPP)

Example 5 - Product Mix Problem:

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Tabular Form		
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	Machine	**Labor**
A	1.5	2.5
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Availability	300 Hrs	240 Hrs

Solution:

Decision variables:

Formulation of Linear Programming Problem(LPP)

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Products	**Resources/unit**	
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A	1.5	2.5
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Solution:

Decision variables:

X_1 = number of units of A manufactured per month.

X_2 = number of units of B manufactured per month.

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B	2.5	1.5
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The Objective Function:

Formulation of Linear Programming Problem(LPP)

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The Objective Function:

$$\text{Max. } Z = 50X_1 + 40X_2$$

Formulation of Linear Programming Problem(LPP)

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Subject to Constraints:

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Solution:

The Objective Function:

$$\text{Max. } Z = 50X_1 + 40X_2$$

Subject to Constraints:

For machine hours: $1.5X_1 + 2.5X_2 \leq 300$

For labor hours: $2.5X_1 + 1.5X_2 \leq 240$

Non-negativity: $X_1, X_2 \geq 0$

Formulation of Linear Programming Problem(LPP)

Example 6 – Mutual Fund Investment decisions:

- Mr. Akshay wants to invest in two mutual funds A & B.
- His total investment is Rs.1000,000 and not more than Rs.700,000 will be invested in a single fund.
- Fund A gives returns of 15% p.a. & fund B gives returns of 20% p.a.
- The risk factor ratings on a scale of 0-10 are, for A-4 and for B-8.
- Minimum desired returns are 17.5% and the maximum tolerable risk factor is 6 for Mr. Akshay.
- Formulate as LPP.

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Solution:

Tabular form

Fund	Returns	Risk factor
A	15% = 0.15	4
B	20% = 0.20	8
	Minimum Returns	Maximum Tolerable Risk = 6
	= 17.5%	
	= 0.175	

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Decision variables:

X_1 = Investment in Rs. in fund A

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Objective function:

Max $Z = 0.15X_1 + 0.2X_2$

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Subject to constraints:

(1)

(2)

(3)

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	Minimum Returns	Maximum Tolerable Risk = 6
	= 17.5%	
	= 0.175	

Decision variables:

X_1 = Investment in Rs. in fund A

X_2 = Investment in Rs. in fund B

Solution:

Objective function:

Max $Z = 0.15X_1 + 0.2X_2$

Subject to constraints:

(1) $X_1 + X_2 = 1000{,}000$ ... (Total investment)

(2) $X_1 \leq 700{,}000$ ... (Maximum investment in a single fund)

(3) $X_2 \leq 700{,}000$

Formulation of Linear Programming Problem(LPP)

Example 6 – Mutual Fund Investment decisions:

- Mr. Akshay wants to invest in two mutual funds A & B.
- His total investment is Rs.1000,000 and not more than Rs.700,000 will be invested in a single fund.
- Fund A gives returns of 15% p.a. & fund B gives returns of 20% p.a.
- The risk factor ratings on a scale of 0-10 are, for A-4 and for B-8.
- Minimum desired returns are 17.5% and the maximum tolerable risk factor is 6 for Mr. Akshay.
- Formulate as LPP.

Tabular form

Fund	Returns	Risk factor
A	15% = 0.15	4
B	20% = 0.20	8
	Minimum Returns	Maximum Tolerable Risk = 6
	= 17.5%	
	= 0.175	

Decision variables:

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Solution:

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(4)

(5)

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Solution:

Objective function:

Max $Z = 0.15X_1 + 0.2X_2$

(4) $4X_1 + 8X_2 \leq 6\ (1000,000)$

Therefore, $4X_1 + 8X_2 \leq 6000,000$ ... (Max risk)

(5) $0.15X_1 + 0.20X_2 \geq 0.175\ (1000,000)$

Therefore, $0.15X_1 + 0.20X_2 \geq 175,000$ ... (Min. returns)

$X_1, X_2 \geq 0$

Formulation of Linear Programming Problem(LPP)

Example 6 – Mutual Fund Investment decisions:

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Objective function:

Max $Z = 0.15X_1 + 0.2X_2$

Subject to constraints:

(1) $X_1 + X_2 = 1000{,}000$ ... (Total investment)

(2) $X_1 \leq 700{,}000$ ... (Maximum investment in a single fund)

(3) $X_2 \leq 700{,}000$

(4) $4X_1 + 8X_2 \leq 6\,(1000{,}000)$

Therefore, $4X_1 + 8X_2 \leq 6000{,}000$ ... (Max risk)

(5) $0.15X_1 + 0.20X_2 \geq 0.175\,(1000{,}000)$

Therefore, $0.15X_1 + 0.20X_2 \geq 175000$, ... (Min. returns)

$X_1, X_2 \geq 0$

Application of OR: 1.4

An Application Vignette

"Prior to its merger with United Airlines that was completed in 2012, **Continental Airlines** was a major U.S. air carrier that transported passengers, cargo, and mail. It operated more than 2,000 daily departures to well over 100 domestic destinations and nearly 100 foreign destinations. Following the merger under the name of United Airlines, the combined airline has a fleet of over 700 aircraft serving up to 370 destinations.

Airlines like Continental (and now under its reincarnation as part of United Airlines) face schedule disruptions daily because of unexpected events, including inclement weather, aircraft mechanical problems, and crew unavailability. These disruptions can cause flight delays and cancellations. As a result, crews may not be in position to service their remaining scheduled flights. Airlines must reassign crews quickly to cover open flights and to return them to their original schedules in a cost-effective manner while honoring all government regulations, contractual obligations, and quality-of-life requirements.

To address such problems, an OR team at Continental Airlines developed a detailed *mathematical model* for reassigning crews to flights as soon as such emergencies arise. Because the airline has thousands of crews and daily flights, the model needed to be huge to consider all possible pairings of crews with flights. Therefore, the model has *millions of decision variables* and *many thousands of constraints*. In its first year of use (mainly in 2001), the model was applied four times to recover from major schedule disruptions (two snowstorms, a flood, and the September 11 terrorist attacks). This led to *savings of approximately* **$40 million.** Subsequent applications extended to many daily minor disruptions as well.

Although other airlines subsequently scrambled to apply operations research in a similar way, this initial advantage over other airlines in being able to recover more quickly from schedule disruptions with fewer delays and canceled flights left Continental Airlines in a relatively strong position as the airline industry struggled through a difficult period during the initial years of the 21st century. This initiative led to Continental winning the prestigious First Prize in the 2002 international competition for the Franz Edelman Award for Achievement in Operations Research and the Management Sciences."

Source: G. Yu, M. Argüello, C. Song, S. M. McGowan, and A. White, "A New Era for Crew Recovery at Continental Airlines," *Interfaces*, **33**(1): 5–22, Jan.–Feb. 2003. (A link to this article is provided on our website, www.mhhe.com/hillier.)

Application of OR: 2.2

An Application Vignette

US Environmental Protection Agency (EPA) Uses Operations Research to Reduce Contamination Risks in Drinking Water

"Security of drinking water in the United States is monitored by the US Environmental Protection Agency (EPA). The agency is responsible for providing information and technical assistance to more than 50,000 water utilities across the country. The distributed physical layout of drinking-water utilities makes them inherently vulnerable to contamination incidents caused by terrorists. A terrorist attack on the drinking-water infrastructure could severely impact the public health and economic vitality of the country. The 9/11 attacks led to a new US government focus on homeland security, especially on reducing vulnerabilities in critical infrastructure. Signi?cant challenges remain in detecting contamination incidents early enough to mitigate both public health and economic impacts.

The EPA has the long-standing responsibility of working with the 50 states, the District of Columbia, tribes and six territories on oversight and implementation of regulations pertaining to the water sector. In 2004, President Bush directed the EPA to 'develop robust, comprehensive and fully coordinated surveillance and monitoring systems that provide early detection and awareness of disease, pest or poisonous agents'. In response to this federal mandate, the EPA's Threat Ensemble Vulnerability Assessment (TEVA) research program is using operations research to develop contamination warning systems (CWSs) that counter threats against the drinking-water infrastructure. A CWS integrates monitoring and surveillance data from multiple detection methods to provide early detection of contamination in drinking-water distribution systems. The TEVA Sensor Placement Optimization Tool (TEVA-SPOT) uses sensor-placement algorithms to design CWSs that reduce the potential impact of contamination incidents through early detection and rapid response. Operations research has played a key role in developing fast algorithms that can design sensor placements for large water networks and that run on standard desktop computers. The EPA has partnered with the American Water Works Association (AWWA) to apply TEVA-SPOT in large US cities to develop CWSs that signi?cantly reduce public health and economic risks. To run the TEVA-SPOT software, the utility must provide speci?c input data: its water-distribution network topology and its utility operational rules, the characteristics of the sensors it wants to deploy, the design basis—the types of contamination incidents it would like the CWS to detect, the performance measure to minimize through optimization, the likely utility response time, and a list of locations that could potentially house sensors. The software simulates all the contamination incidents described in the design basis, calculates the potential impacts of those incidents, and selects an optimal sensor design using operations research models that reduces those impacts. By simulating the incidents and calculating the impacts over space and time, the optimization routine can determine the best locations for sensors and the times at which they will detect incidents. This approach based on operations research methodologies allows utilities to use early detection and rapid response to maximize the potential reduction in consequences of contamination incidents.

Application of OR: 2.2

Through the TEVA program, operations research has changed the direction of water security in the United States. Online monitoring has become the accepted technology for reducing contamination risks in drinking-water systems. By demonstrating that online monitoring is more cost-effective than routine sampling and analysis programs, 75 percent cost reduction in utility CWS deployments was achieved, resulting in savings of millions of dollars at each utility. CWSs were designed for nine of the largest US water systems, which collectively serve more than 17 million customers. By collaborating with industry, academia and government, the ideas and software tools developed as part of this program have been widely adopted and accepted by the water industry. Widespread application of these new systems will significantly reduce the risks associated with catastrophic contamination incidents: the median-estimated fatalities reduction for the nine utilities already studied is 48 percent; the corresponding economic-impact reduction is over $19 billion. Because of this operations research program, online monitoring programs, such as a CWS, are now the accepted technology for reducing contamination risks in drinking water."

Source: Murray, Regan, William E. Hart, Cynthia A. Phillips, Jonathan Berry, Erik G. Boman, Robert D. Carr, Lee Ann Riesen et al. 'US Environmental Protection Agency uses operations research to reduce contamination risks in drinking water'. *Interfaces* 39, no. 1 (2009): 57–68.